The Rise and Fall of Investment: Rethinking Q theory in Equilibrium

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Abstract

The classic Q theory of investment is commonly interpreted to assert that marginal Q, synonymous to the marginal value of capital, is the sufficient statistic for investment. That is because Q-theory is purely demand-based in the sense that variations in investment are fully driven by those in the demand for investment.

This paper provides an exposition of how shocks to the supply of investment drive the joint dynamics of investment and Q. In absence of shocks to the marginal cost of investment (i.e., the supply of investment), shocks to the marginal value of investment (i.e., the demand for investment) determine both equilibrium investment and Q, resulting in a conventionally expected monotonic relation along the constant upward-sloping investment supply curve. In presence of non-trivial shocks to the marginal cost of investment, however, there is no longer a one-to-one relation between investment and Q. In essence, Q is to investment as price to quantity in any demand-supply system. This paper theoretically demonstrates that, in a general dynamic model of investment, shocks to the investment demand induce a positive comovement between investment and Q when the marginal cost of investment is monotonically increasing, while shocks to the investment supply induce a negative comovement of investment and Q when investment is sufficiently inelastic to supply shocks. The elasticity of investment to demand and supply shocks critically depends on their respective persistence. This paper shows with numerical simulations that the correlation between investment and marginal/average Q critically depends on the relative volatility of and the persistence of supply shocks. A modest level of volatility of supply shocks is able to generate low or even negative correlations between investment and Q.

In summary, one should rethink from an equilibrium view the relation between investment and Q, both of which are simultaneously determined by shocks to both investment demand and supply.

Keyword: Investment, Q theory, investment demand and supply, adjustment cost

Tobin (1969): The rate of investment—the speed at which investors wish to increase the capital stock—should be related, if to anything, to q, the value of capital relative to its replacement cost.

1 Introduction

Investment serves as a pivotal driver of economic growth, plays a central role in business cycle dynamics, and stands as a crucial target for both monetary and fiscal policy design. Therefore, it is important to understand the behavior of investment. What does determine investment? This enduring question has garnered substantial research attention and efforts in both macroeconomics and corporate finance.

Rooted in thoughts of Keynes (1936)[1], Grunfeld (1960)[2], and Tobin (1969), the Q theory of investment emphasizes the central role of the market value of capital relative to its replacement costs in driving investment decisions. Subsequently formalized by Lucas Jr and Prescott (1971), Mussa (1977), A. Abel (1979), Yoshikawa (1980), Q-theory says that the optimal rate of investment is such that marginal Q, the ratio of the market value of an additional unit of capital to its replacement cost, equals the marginal cost of investment. Furthermore, Hayashi (1982) shows that, under the condition of constant returns-to-scale technology, marginal Q equals average Q—the ratio of the market value of existing capital to its replacement cost, commonly referred to as Tobin's Q. This equivalence result has deeply influenced the study of both aggregate and corporate investment for more than three decades, despite a long-standing consensus about its empirical limitations.[3]

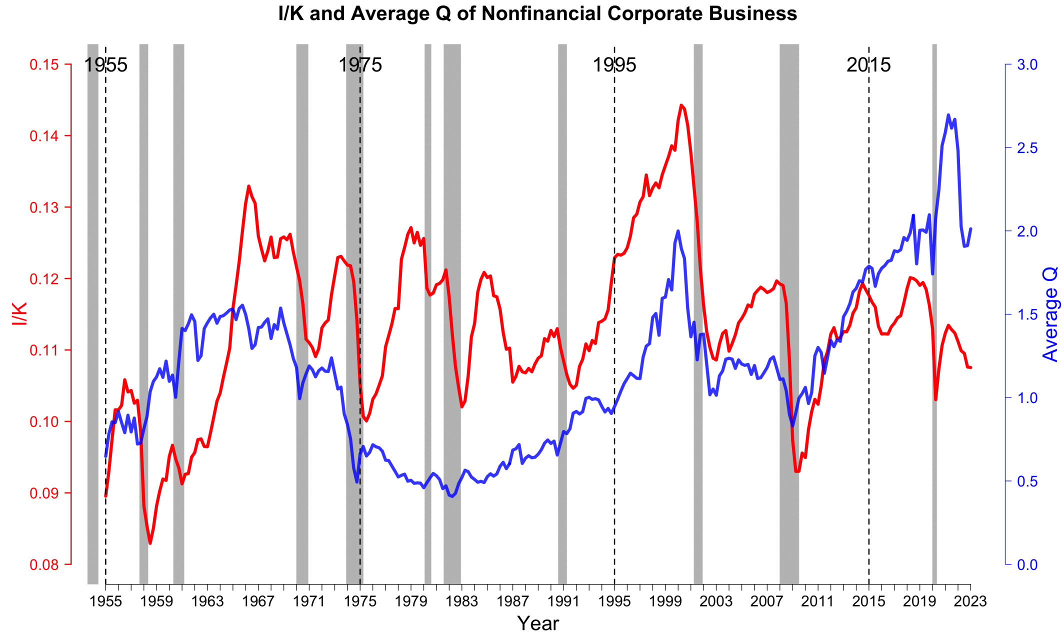
Q-theory is commonly interpreted to assert that Q is the sufficient statistic for investment, a perspective we term *the fundamental view*. Under the fundamental view, the following empirical hypotheses are formulated, in ascending order of assertion, (1) that investment is strongly

[1] Keynes (1936): *For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased; whilst there is an inducement to spend on a new project what may seem an extravagant sum, if it can be floated off on the Stock Exchange at an immediate profit. Thus certain classes of investment are governed by the average expectation of those who deal on the Stock Exchange as revealed in the price of shares, rather than by the genuine expectations of the professional entrepreneur.*

[2] Grunfeld (1960): *... that the role of profits (in explaining investment behavior) is probably that of a surrogate variable... there are other variables which reflect these forces better... The principal variable of this type is the "market value of the firm", that is, the value placed upon the firm by the securities markets. When taken in conjunction with an estimate of the replacement value of the physical assets of the firm, this variable appears to be a sensitive indicator of the expectations upon which investment decisions are based.*

[3] The disappointing empirical performance is succinctly summarized by Philippon (2009), "The investment equation fits poorly, leaves large unexplained residuals correlated with cash flows, and implies implausible parameters for the adjustment cost function".

Figure 1: Investment rate and average Q of nonfinancial corporate business



Note: The investment rate is calculated as the ratio of gross fixed investment to one-period lagged fixed assets. The numerator of investment rates, gross fixed investment, is measured by NFCB gross fixed investment in "nonresidential structures, equipment, and intellectual property products (NRSEIP) (FA105013005Q). The denominator is the one-period lagged NFCB nonresidential fixed assets in NRSEIP measured on the basis of current costs (FL105013865Q). Average Q is calculated as the ratio of the market value of installed capital to the replacement cost of installed capital. The numerator, the market value of NRSEIP, is measured by NFCB market value estimate of nonfinancial assets (FL102010405Q), less nonfinancial assets that are not NRSEIP. NFCB market value estimate of nonfinancial assets (FL102010405Q) is the sum of corporate equities (FL103164103Q), foreign direct investment in U.S (FL103192105Q), total liabilities (FL104190005Q), less total financial assets (FL104090005Q). Nonfinancial assets other than NRSEIP includes current costs of inventories excluding IVA (FL105020015Q), current costs of residential equipment (FL105012265Q), market value of residential real estate (FL105035023Q), book value of vacant land (FL105010103Q).NFCB nonfinancial assets (FL102010005Q) includes real estate at market value (FL105035005Q), equipment, current cost basis (FL105015205Q), nonresidential intellectual property products, current cost basis (FL105013765Q), and inventories excluding IVA, current cost basis (FL105020015). We remove components under NFCB nonfinancial assets that are not NRSEIP from NFCB market value estimate of nonfinancial assets. The denominator, the replacement costs of installed capital, is measured by contemporaneous NFCB nonresidential fixed assets in NRSEIP measured on the basis of current costs (FL105013865Q).

positively correlated with Q, (2) that investment is fully explained by Q, subsuming any additional variables, and (3) that investment is causally determined by Q.

The disappointing empirical performance of the investment-Q regression has been, under the fundamental view, commonly attributed to the lack of measures of the true marginal Q that is difficult, if not entirely impossible, to be measured empirically. For example, average Q exceeds marginal Q when firms enjoy monopoly rents due to DRS technology or imperfect competition (Lindenberg and Ross (1981)). Average Q fails to capture marginal Q for the specific type of capital when capital goods are heterogeneous and cannot be simply summed up (Hayashi and Inoue (1990)). Average Q overstates marginal Q for physical capital when firm values are derived from intangible capital (Hall (2001); Peters and Taylor (2017)). Average Q

can also differ from true marginal Q in presence of empirical measurement errors (Erickson and Whited (2000)). In other words, Q-theory does not fail, but those overly simplistic auxiliary assumptions fail, rendering Q-theory barely empirically testable.

This paper proposes *the equilibrium view*, which posits that investment and Q are simultaneously determined and driven by common underlying state variables. Under the equilibrium view, Q is neither the causal determinant of nor the sufficient statistic for investment. Q is to investment is as price is to quantity in any demand-supply system. The joint dynamics of investment and Q critically depends on the nature of shocks, whether originated from the demand or supply side.

First, we transparently elucidate the intuition and derive some generalized propositions. Analogous to quantity and price, investment and Q are jointly determined at equilibrium by the intersection of the marginal benefit of investment (i.e., investment demand) and the marginal cost of investment (i.e., investment supply). Positive shocks to expected profitability drive up the marginal benefit of investment and thereby raise the demand for investment, while positive shocks to adjustment costs drive up the marginal cost of investment and thereby reduce the supply of investment. Thus, a positive shock to adjustment costs is a negative shock to the investment supply.[4] We theoretically demonstrate that, in a dynamic and stochastic model of investment under general conditions, shocks to the investment demand induce a positive comovement between investment and Q so long as the marginal cost of investment is strictly monotonic. In contrast, shocks to the investment supply induce a negative comovement of investment and Q if and only if investment is sufficiently inelastic to supply shocks. Consequently, in presence of shocks to both investment demand and investment supply, the correlation between investment and Q is a priori ambiguous, and the investment-Q regression is endogenous.

Second, we analyze the special case of constant returns-to-scale (CRS) technology. We analytically solve, to a first-order approximation, elasticities of investment to both demand and supply shocks that are stationary AR(1) processes. We show that, the elasticity of investment to demand shocks is non-negative and monotonically increasing in the persistence of demand shocks, while the elasticity of investment to supply shocks is negative and monotonically decreasing in the persistence of supply shocks. In addition, the elasticity of investment to supply shocks is sufficiently elastic that both investment and Q decrease given a negative

[4]We delay the discussion of the nature of shocks to adjustment costs to the end of the introduction.

supply shock (i.e., a positive shock to adjustment costs) so long as supply shocks are positively autocorrelated. Geometrically, this sufficiently high (and negative) elasticity means that a positive shock to adjustment costs simultaneously steepens the investment supply curve that is upward sloping and shifts downward the investment demand curve that is flat. Intuitively, when supply shocks are i.i.d., they affect current investment decisions only and leave the investment demand curve and conditional future investment decisions unchanged. On the other extreme when supply shocks are permanent, a positive shock to adjustment costs permanently increases the cost of investment for today and the future, resulting in a permanently lower level investment, holding demand unchanged. That means investment is so elastic to permanent supply shocks that the reduction in investment more than offsets the increase in adjustment cost shock, resulting in a permanently lower marginal Q. The case of positively autocorrelated supply shocks is qualitatively similar to the case of permanent supply shocks.

Finally, we numerically solve and simulate the model under the condition of DRS technology. This case is more interesting as it allows investment and Q to move in opposite directions subsequent to supply shocks. The joint dynamics of investment and Q crucially depends on the conditional volatilities and persistence of both demand and supply shocks. First, the correlation between investment and marginal Q crucially depends on both the relative volatility and the persistence of supply shocks. As supply shocks become more volatile and persistent, the correlation between investment and marginal Q below one and, under some values, below zero. Therefore, the hypothesis that marginal Q, if perfectly measured, should always be perfectly correlated with investment fails when supply shocks are sufficiently volatile and persistent. Second, the correlation between investment and average Q is dominantly determined by the relative volatility of supply shocks and barely affected by the persistence of supply shocks. The correlation between investment and average Q can go arbitrarily low and even become negative as well. In fact, it does not take a very volatile supply shock to make both correlations negative. Therefore, the empirically observed low correlation between investment and average Q can be attributed to a relatively volatile supply shock, without resorting to the existence of intangible capital.

Investment Supply Shocks. What are shocks to the supply of investment really? There are two types of investment supply shocks. The first kind refers to shocks to the relative price of investment goods, commonly known as investment-specific technology (IST) shocks, dating back to Greenwood, Hercowitz, and Huffman (1988) and Greenwood, Hercowitz, and Krusell (1997).

IST shocks can arise from the improvement in the productivity of the investment goods sector or in the efficiency of newly produced investment goods, and they have important implications for economic growth and social welfare. IST shocks, however, do not affect the relation between investment and Q. In other words, in presence of shocks to the relative price of investment goods, Q remains the sufficient statistic for investment. This is because Q (either marginal or average) is defined as the ratio of the (either marginal or total) market value of capital to its replacement costs, instead of its quantity and, therefore, already captures variations in the relative price of investment goods. Equivalently, Q-theory models almost always normalize the price of investment goods to one so that all market prices and values are denominated in units of investment goods.

The other kind, which we emphasize in this paper, refers to shocks to adjustment costs of investment. Adjustment cost shocks, as illustrated in this paper, shift and bend the supply curve of investment, fundamentally changing the relation between investment and Q. But, what are really adjustment costs and shocks to them? There are two classes of interpretations. First, as in Mussa (1977), Chirinko (1993), Cooper and Haltiwanger (2006), adjustment costs are costs associated with adjusting the capital stock that could arise from the installation of new capital, the interruption of current production, retraining of workers, etc. In this sense, adjustment costs are opportunity costs associated with the installation of capital that transform new uninstalled investment goods into productive capital.[5] In other words, adjustment costs create a wedge between the price of uninstalled investment goods and that of installed capital, making installed capital more than valuable.

Adjustment costs can also be thought of as financing costs of investment in some models. Bolton, Chen, and Wang (2011) shows that the effective marginal cost of investment is the product of the standard marginal cost of investment with convex adjustment costs and the marginal cost of financing. Similar setups are also present in *gomes2001*, Ottonello and Winberry (2020). Chari, Kehoe, and McGrattan (2007) show that an economy with credit market frictions as in Beranke and Gertler (1989) and Carlstrom and Fuerst (1997) is equivalent to a growth model

[5] A WSJ article, titled "First Big U.S. EV-Battery Plant Offers Lessons as Industry Springs Up", on Feb 25th 2023 told the story of difficulties of Panasonic building factories and manufacturing EV battery in the U.S. For example, on training workers: "One of the biggest issues is training workers in the finicky art of battery making", "...American workers' hands were sometimes too big to efficiently operate machinery made in Asia". On installation: "Also, equipment can't necessarily be shipped from Asia and plopped onto an American assembly line, given U.S. safety regulations and different operating conditions, while equipment customized for the U.S. is in short supply." As a result, "Boosting production took a year or two more than expected because of issues such as training workers without battery experience and adapting equipment and production processes to them..."

with an time-varying investment wedge. Wang and Wen (2012) show that collateral constraint as in Kiyotaki and Moore (1997) at the firm-level can give rise to the convex adjustment cost at the aggregate level.

In this paper, we remain agnostic about the source of shocks to adjustment costs but emphasize the importance of supply shocks in driving the joint dynamics of investment and Q. Regardless of the source of adjustment costs shocks, either mechanical or financial, the ultimate effect is shifting or bending the marginal cost curve of investment such that investment and Q could potentially move in any direction.

Related Literature. We are certainly not the first to become aware of the simultaneity issue of investment and Q. For example, Clark et al. (1979) directly points out, "...both investment and the ratio of market value to replacement cost react to the same state of long-run expectations about future output and prices. When real capital is expected to be profitable in the future, both investment and Q rise", suggesting that investment and Q are simultaneously driven by the expected profitability. Abel and Blanchard (1986) note that simultaneity problems arise if Q depends on current variables. We are neither the first study to consider supply shocks in the model. For example, Hayashi and Inoue (1990), Gilchrist and Himmelberg (1995), Erickson and Whited (2000) all include supply shocks in their models.[6] Aware of the endogeneity arising from the existence of adjustment cost shocks[7], they address this issue by different means.[8] They argue that the presence of adjustment cost shocks can be circumvented econometrically under some generic, if not strong, assumptions. Instead of getting around the endogeneity issue, we argue that, under general conditions, adjustment cost shocks have non-trivial implications for the joint dynamics of investment and Q and that empirical hypotheses about investment and Q should be reformulated in presence of adjustment costs shocks.

Our paper is closely related to Justiniano, Primiceri, and Tambalotti (2010) and Justiniano, Primiceri, and Tambalotti (2011) (JPT henceforth). JPT (2010) shows that, at business cycle frequencies, investment shocks account for more than 80 percent of those in investment, while also contributing to 50 percent of the variations in output and almost 60 percent of those in

[6] We discuss different specifications of adjustment costs by them and others in details later.

[7] Hayashi and Inoue (1990): "Q is a function of technology shock and hence is econometrically endogenous". Erickson and Whited (2000) also noted that the endogeneity arising from the existence of adjustment cost shocks is one of two major criticisms of the investment-Q regression.

[8] Hayashi and Inoue (1990) argue that, under some conditions, one can instrument contemporaneous Q by its lagged (or even future) counterparts. Gilchrist and Himmelberg (1995) construct instruments for Q using VAR forecasts from observed fundamentals as in Abel and Blanchard (1986). Erickson and Whited (2000) argues that the endogeneity is absent under some strong assumptions.

labor hours. In an augmented model, JPT (2011) demonstrates that incorporating IST shocks and identifying them using the relative price of investment goods barely change the quantitative role of investment shocks in business cycle fluctuations. We differ from JPT in three important ways. First, we focus entirely on investment and the joint dynamics of investment and Q, whereas JPT are concerned with the real business cycle. Second, we utilize a partial equilibrium model that is as minimal and general as possible so that it can be incorporated into larger models more seamlessly. In contrast, JPT (2010, 2011) provide fully-fledged general equilibrium models. Finally, a technical difference is that we specify the adjustment cost as a function of the investment-to-capital ratio instead of the investment growth.

Our paper is also closely related to Gala, Gomes, and Liu (2020) (GGL henceforth). We share the same view with GGL that the optimal policy of investment is ultimately a function of underlying exogenous state variables, which, as a whole, are true summary statistics for investment. This view shifts the focus of investment from Q to relevant state variables. GLL non-parametrically estimates the empirical policy function and their results provide an important reference for subsequent work. We differ from GLL in two important ways. First, we demonstrate both theoretically and numerically how shocks of different natures affect the dynamics of investment, whereas GLL approaches state variables empirically and remains agnostic about the ultimate source of state variables. Second, more importantly, we emphasize the role of supply shocks, whereas in GLL empirically identified state variables (i.e., firm size and productivity) are in fact demand-side variables in our framework.

The remainder of the paper is organized as follows. Section 2 derives the generic Q-theory model and reviews common interpretations of Q-theory, termed as "the fundamental view". Section 3 introduces "the equilibrium view", first illustrated in a simple static two-period model and further analyzed in a dynamic and stochastic model. Section 4 analyzes the case of constant returns-to-scale (CRS) technology using log-linear approximations. Section 5 numerically solves the case of decreasing returns-to-scale (DRS) technology and analyze the correlation between investment and marginal/average Q using simulations. Section 6 concludes.

2 Basics of Q Theory

2.1 Setup

2.1.1 Profit function

Consider a firm that uses physical capital, K_t, and labor, N_t, to produce non-storable output, Y_t. The production function is specified to be Cobb-Douglas with unit elasticity of substitution between capital and labor

$$Y_t = X_t(K_t^{\gamma} N_t^{1-\gamma})^s \tag{1}$$

where X_t is the total factor productivity (TFP), $0 < s \leq 1$ is the degree of returns to scale, and $0 < \gamma < 1$. The production function exhibits constant returns to scale (CRS) when $s = 1$ and decreasing returns to scale (DRS) when $0 < s < 1$.

The firm can be subject to perfect or imperfect competition, which we model in a reduced-form manner by specifying the inverse demand function.[9] The price of output is given by

$$P_t^C = \left(\frac{Y_t}{H_t}\right)^{-\frac{1}{\omega}} \tag{2}$$

where $\omega > 0$ is the elasticity of demand to price. The firm faces perfect competition when demand is perfectly elastic, $\omega = \infty$, meaning that the firm's production decision exerts no externality on the market price. The firm enjoys some market power when demand is finitely elastic, $0 < \omega < \infty$, meaning that the firm should internalize the impact of its quantity of output on the market price. H_t determines the relative location of the demand curve, and, more precisely, $h_t \equiv \log(H_t)$ is the intercept of the log demand function, $y_t = h_t - \omega p_t^C$, where lowercase letters denote their logarithm counterparts.[10] Shocks to H_t shift the demand curve.

The capital is costly adjustable (imperfectly variable) in a sense to be defined shortly, while labor is costlessly adjustable (perfectly variable).[11] The capital is owned by the firm and accumulated by investment, while labor is rented at the market wage rate. With the current stock of physical capital K_t, the firm chooses labor N_t to maximize its operating profit for a given productivity X_t, demand H_t, and wage rate W_t. The operating profit function, which is revenues

[9] This functional form can be derived from a CES utility function, in which the elasticity of substitution is ω.

[10] Throughout the paper, we use uppercase letters to denote levels and lowercase letters to denote their logarithm counterparts.

[11] It can be any other perfectly variable inputs other than labor.

less labor costs, is given by

$$\Pi(A_t, K_t) := A_t^{1-\nu} K_t^{\nu} \equiv \max_{N_t} \ P_t^C Y_t - W_t N_t \tag{3}$$

where A_t is a geometric average of demand shocks H_t, productivity shocks X_t, and wage shocks W_t, and ν is a composite parameter. They are

$$A_t := \left[(1-\theta)\theta^{\frac{\theta}{1-\theta}}\right] H_t^{\frac{1}{\omega(1-\theta)}} X_t^{\frac{\omega-1}{\omega(1-\theta)}} W_t^{\frac{-\omega\theta}{\omega(1-\theta)}}; \quad \nu := \frac{\gamma s\left(1-\frac{1}{\omega}\right)}{1-(1-\gamma)s\left(1-\frac{1}{\omega}\right)} \tag{4}$$

where $\theta := (1-\gamma)s\left(1-\frac{1}{\omega}\right)$ is also a composite parameter. In the special case of $\nu = 1$, that is when $\omega = \infty$ and $s = 1$, the profit function is given by $\Pi_t = A_t K_t$, where $A_t := \gamma(1-\gamma)^{\frac{1-\gamma}{\gamma}} X_t^{\frac{1}{\gamma}} W_t^{\frac{\gamma-1}{\gamma}}$ is both the marginal and average profitability. The setup of the profit function, from (**??**) to (**??**), following exactly Abel and Eberly (2011) with only notational differences, incorporates monopoly rents in a convenient manner.

The operating profit function exhibits CRS ($\nu = 1$), if and only if the firm is subject to perfect competition ($\omega = \infty$) and the production function is CRS ($s = 1$). The operating profit function exhibits DRS ($\nu < 1$), if the firm enjoys some market power ($0 < \omega < \infty$) or if the production function is DRS ($s < 1$). For simplicity, we will use the profit function (**??**) for the rest of the paper, because (1) the firm does not accrue rents from hiring perfectly variable labor and (2) profits rather than revenues are ultimately distributed or reinvested. In fact, for the exposition of Q theory, one can directly specify the profit function as (**??**) or simply as $A_t K_t^{\nu}$ without loss of generality.[12]

2.1.2 Investment adjustment cost

The capital stock depreciates by a fraction of δ every period and is replenished by investment.

$$K_{t+1} = (1-\delta)K_t + I_t \tag{5}$$

To invest I_t units, it costs the firm $P_t^I I_t$ to purchase investment goods, where P_t^I is the price of investment goods. We normalize P_t^I to one, equivalent to scaling all variables by P_t^I, so that the numeraire in the model is the investment good. In addition to purchase costs of investment goods, the firm also incurs investment adjustment cost Φ_t.

[12] While, in this case, the profit function resembles the "AK" production function in the literature of endogenous growth, they are not the same. In Q theory, what matters is the ultimate profit function.

The investment adjustment cost represents costs that are associated with the transformation of investment goods into productive capital. For example, installing new capital may require plant restructuring (Cooper and Haltiwanger (2006)), worker retraining (Atkin et al. (2017)), organization restructuring (Bresnahan, Brynjolfsson, and Hitt (2002)), and regulation compliance (Kalmenovitz (2023)), resulting in production interruption, loss of output, or additional costs. The investment adjustment cost was originally introduced to build some degree of "capital fixity" in the short-run, as illustrated by Lucas Jr (1967). Otherwise, given a permanent shock to the profitability, the firm immediately adjusts its capital stock to the new long-run level. While there are a number of alternative ways to achieve the same effect, introducing the adjustment cost is arguably the most common and convenient modeling approach.

Following Eisner and Strotz (1963), Lucas Jr (1967), Gould (1968), and Treadway (1969), the investment adjustment cost is commonly specified as an increasing and convex function of investment, with $\Phi_I > 0$, $\Phi_{II} > 0$, to capture the increasing difficulty of the formation of new productive capital.[13] It is also often specified as a decreasing function of the capital stock, to capture the base effect that it is more costly for a firm with smaller capital stock to install the same amount of capital goods. A commonly used parameterization is given by

$$\Phi(C_t, I_t, K_t) = \frac{C_t}{\eta + 1}\left(\frac{I_t}{K_t}\right)^{\eta+1} K_t \tag{6}$$

where $C_t > 0$ is a stochastic scalar controlling the size of adjustment costs, and $\eta \geq 0$ is the curvature of adjustment costs. This parameterization exhibits CRS in I_t and K_t, i.e., $\Phi_t = I_t \frac{\partial \Phi_t}{\partial I_t} + K_t \frac{\partial \Phi_t}{\partial K_t}$. The stochastic scalar C_t distinguishes this model from the standard Q theory, and its implications will be the focus of this study.

2.1.3 Payout and timing

The firm is assumed to be fully equity-financed. The firm distributes remaining operating profits after costs of investment to shareholders. The dividend is given by

$$D_t = \Pi_t - I_t - \Phi_t \tag{7}$$

[13] An alternative and equivalent approach to modelling costly adjustment is to specify the law of motion for physical capital as $K_{t+1} = (1-\delta)K_t + \Phi(I_t, K_t)$, where $\Phi(\cdot)$ is a weakly concave function of investment, to capture the decreasing marginal efficiency of investment, i.e., changing the capital stock rapidly is more costly than changing it slowly. This specification can be interpreted as some investment goods are lost during the installation. See Uzawa (1969), Lucas Jr and Prescott (1971) for early contributions. The specification was adopted by Hayashi (1982), Baxter and Crucini (1993), Jermann (1998), Kaltenbrunner and Lochstoer (2010).

where a negative value of dividend refers to issuing new equity.

The timing of the model is standard as follows. The firm starts with capital K_t and observes the realized profit Π_t at the beginning of period t, and then decides its investment I_t and dividend D_t. However, realizations of both profit and investment are not observable by econometricians or investors until the end of the period. The new investment goods only becomes productive at the beginning of period $t+1$. In period $t+1$, the firm owns productive capital stock, $K_{t+1} = (1-\delta)K_t + I_t$, and goes through the same sequence of events as above.[14]

2.2 Optimality Condition

At time t, taking as given the demand shifter H_t, the productivity X_t, the wage rate W_t, the adjustment cost scalar C_t, and the stochastic discount factor (SDF), the firm maximizes its value, which is the present value of all dividends, by optimally choosing labor and investment.

$$\max_{\{N_\tau,\, I_\tau,\, K_{\tau+1}\}_{\tau=t}^{\infty}} \quad \mathbb{E}_t\left[\sum_{\tau=t}^{\infty} M_{t,\tau}\left(P_\tau^C Y_\tau - W_\tau N_\tau - I_\tau - \Phi_\tau\right)\right]$$

where $M_{t,\tau}$ is the exogenous SDF from time t to τ with $M_{t,t} = 1$.

Alternatively, the optimization problem can be conveniently formulated in a recursive manner. Denote $\mathcal{S}_t = (K_t, A_t, C_t)$ as the vector of state variables and $V(\mathcal{S}_t)$ as the value function at time t, the Bellman equation of the firm is given by

$$\begin{aligned} V(\mathcal{S}_t) &= \max_{\{I_t, K_{t+1}\}} \left\{D_t + \mathbb{E}_t\big[M_{t+1}V(\mathcal{S}_{t+1})\big]\right\} \\ s.t. \quad D_t &= \Pi(A_t, K_t) - I_t - \Phi(C_t, I_t, K_t) \\ K_{t+1} &= (1-\delta)K_t + I_t \end{aligned}$$

where M_{t+1} is the SDF from time t to $t+1$. Note that, when the profit function exhibits CRS ($\nu = 1$), the strict convexity is required of the adjustment cost function Φ_t (i.e., $C_t > 0,\ \eta > 0$) to make the optimization problem concave and ensure the existence of interior solutions.

The first-order condition with respect to I_t is given by

$$MCI_t := 1 + \frac{\partial \Phi_t}{\partial I_t} = Q_t \tag{8}$$

[14] By the timing convention, K_{t+1} is deterministic in period t for the firm, as the investment decision is made in period t and the depreciation rate is deterministic.

which shows that at optimum the marginal cost of investment (MCI_t) equals Q_t, where Q_t denotes the Lagrange multiplier on the law of motion for capital and is the shadow price of installed capital at time t.

The first-order condition with respect to K_{t+1} is given by

$$Q_t = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] =: MBI_t \tag{9}$$

which shows that Q_t equals the marginal benefit of investment MBI_t, which is the expected discounted value of marginal payoff $MPI_{t+1} := \frac{\partial V_{t+1}}{\partial K_{t+1}}$ accrued to an additional unit of investment. Therefore, the marginal benefit of investment is often termed marginal Q in the literature, and both terms are used inter-exchangably throughout the paper.[15] In addition, it is worth noting that, rigorously speaking, marginal Q_t equals the MBI_t scaled by P_t^I, i.e., $Q_t := MBI_t/P_t^I$, when the price of investment goods P_t^I is not normalized to one.

Equating (**??**) and (**??**), we have

$$MCI_t = 1 + \frac{\partial \Phi_t}{\partial I_t} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] = MBI_t \tag{10}$$

which says that at optimum the the marginal cost equals the marginal benefit of investment (marginal Q). Equivalently, the optimal investment policy equates the marginal cost and the marginal benefit of investment. The intuition here is identical to that of static profit maximization in (**??**), in which the optimum is attained when the marginal cost equals the marginal revenue or, equivalently, when the quantity of output equates the marginal cost and the marginal revenue. The difference, however, is that the marginal benefit is the present value of risky future payoffs, whereas the marginal revenue is deterministic and static in nature. Therefore, investment decisions are naturally more complex and, intuitively, should be related to some forward-looking measures such as valuations ratios as Q-theory suggests.

Using the explicit functional form and holding C_t constant, we can rewrite (**??**) as follows.

$$IK_t = \left(\frac{Q_t - 1}{C}\right)^{\frac{1}{\eta}} \tag{11}$$

which says that the optimal investment rate is positively related to marginal Q. While the theory is theoretically intuitive and simple, it does not directly deliver empirically testable pre-

[15]We note that Campbell (2018) (p.210) terms the marginal payoff $MPI_t := \frac{\partial V_t}{\partial K_t}$, as marginal Q, a terminology which we think is inconsistent with the literature.

dictions as marginal Q is unobservable. Hayashi (1982) shows that marginal Q equals average Q, if the technology, including the profit function and the adjustment cost function, exhibits constant returns to scale.

$$\text{Marginal } Q_t := \frac{\partial P_t}{\partial K_{t+1}} \equiv \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] = \mathbb{E}_t\left[M_{t+1}\frac{V_{t+1}}{K_{t+1}}\right] \equiv \frac{P_t}{K_{t+1}} =: \text{Average } Q_t \tag{12}$$

where $P_t := V_t - D_t$ is the ex-dividend value of the firm. The proof of the middle equality will be shown shortly below. The average Q can be empirically measured by the ratio of the market value of installed capital to its replacement cost, which is exactly consistent with the intuition of Tobin's Q.

To prove the equality, we first use the envelope theorem to expand the marginal payoff of investment (MPI).

$$\frac{\partial V_t}{\partial K_t} = \frac{\partial \Pi_t}{\partial K_t} - \frac{\partial \Phi_t}{\partial K_t} + (1-\delta)\mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] \tag{13}$$

The marginal payoff of investment (MPI) has three terms. The first term is the marginal profit of an additional unit of capital. The second term represents the marginal reduction in adjustment costs of an additional unit of capital for a given level of investment. The last term is the marginal continuation value of an additional unit of capital net of depreciation. In other words, the marginal payoff has two components, the marginal dividend payout, which are the first two terms combined, and the marginal capital gain, which is the last term.

Combing (**??**) and (**??**), we finally obtain the necessary condition of optimality.

$$1 + \frac{\partial \Phi_t}{\partial I_t} = \mathbb{E}_t\left[M_{t+1}\left(\frac{\partial \Pi_{t+1}}{\partial K_{t+1}} - \frac{\partial \Phi_{t+1}}{\partial K_{t+1}} + (1-\delta)\left(1 + \frac{\partial \Phi_{t+1}}{\partial I_{t+1}}\right)\right)\right] \tag{14}$$

To show (**??**), the equality of marginal and average Q under CRS, multiplying K_{t+1} on both sides of (**??**), we have

$$
\begin{aligned}
\left(1+\frac{\partial \Phi_t}{\partial I_t}\right)K_{t+1} &= \mathbb{E}_t\left[M_{t+1}\left(\frac{\partial \Pi_{t+1}}{\partial K_{t+1}}K_{t+1} - \frac{\partial \Phi_{t+1}}{\partial K_{t+1}}K_{t+1} + (1-\delta)\left(1+\frac{\partial \Phi_{t+1}}{\partial I_{t+1}}\right)K_{t+1}\right)\right] \\
&= \mathbb{E}_t\left[M_{t+1}\left(\frac{\partial \Pi_{t+1}}{\partial K_{t+1}}K_{t+1} - \frac{\partial \Phi_{t+1}}{\partial K_{t+1}}K_{t+1} + \left(1+\frac{\partial \Phi_{t+1}}{\partial I_{t+1}}\right)(K_{t+2}-I_{t+1})\right)\right] \\
&= \mathbb{E}_t\left[M_{t+1}\left(\frac{\partial \Pi_{t+1}}{\partial K_{t+1}}K_{t+1} - I_{t+1} - \frac{\partial \Phi_{t+1}}{\partial I_{t+1}}I_{t+1} - \frac{\partial \Phi_{t+1}}{\partial K_{t+1}}K_{t+1} + \left(1+\frac{\partial \Phi_{t+1}}{\partial I_{t+1}}\right)K_{t+2}\right)\right] \\
Q_t K_{t+1} &= \mathbb{E}_t\left[M_{t+1}\Big(\Pi_{t+1} - I_{t+1} - \Phi_{t+1} + Q_{t+1}K_{t+2}\Big)\right] \\
&= \mathbb{E}_t\left[M_{t+1}\Big(D_{t+1} + Q_{t+1}K_{t+2}\Big)\right] \\
&= \mathbb{E}_t\left[\sum_{\tau=t+1}^{\infty} M_{t,\tau}D_\tau\right] = V_t - D_t \equiv P_t
\end{aligned}
$$

where the second equality substitutes $(1-\delta)K_{t+1}$ by $(K_{t+2}-I_{t+1})$, the fourth equality uses the CRS condition, the sixth equality iterates the equation forward by constantly substituting $(1+\frac{\partial \Phi_{t+1}}{\partial I_{t+1}})K_{t+2}$, and the last equality holds assuming the transversality condition, $\lim_{T\to\infty}\mathbb{E}_t[M_T(A_T+\frac{\partial \Phi_T}{\partial I_T})K_T]=0$, holds. Finally, we have Marginal Q_t = Average $Q_t = P_t/K_{t+1}$.

This elegant theoretical result of Hayashi (1982) has consequently sparked a large literature on studying the empirical relation between investment and Q.

2.3 Conventional Views

The investment-Q relation that is frequently invoked in the literature assumes CRS ($\nu = 1$) and a quadratic adjustment cost function ($\eta = 1$) and is given as follows.

$$
IK_t = \frac{Q_t - 1}{C}; \quad Q_t = \frac{P_t}{K_{t+1}} \tag{15}
$$

The "*fundamental view*", based upon this simple equation, interprets Q theory as that investment is directly and positively related to Tobin's Q. Specifically, the fundamental view, in its weak form, expects that investment and Q are strongly positively correlated.[16] The semi-strong form of the fundamental view expects that, in the regression of investment rates on Q, investment is fully explained by Q, which subsumes any other variables added to the regression, and

[16]For example, see Andrei, Mann, and Moyen (2019), which is titled "Why did the Q theory of investment start working?". They regard a tight relation between investment and Q as a necessary ingredient of the empirical success of Q theory. Also, Crouzet and Eberly (2023) starts by noting that, "In recent years, US investment has been lackluster, despite rising valuations", and views the divergence of investment and valuations as "apparently contradictory".

the slope estimator recovers the inverse of the adjustment cost scalar ($1/C$).[17] The semi-strong form is the most common interpretation of Q theory, and it is stronger than the weak form in requiring that Q has the exclusive explanatory power for investment. In its strong form, the fundamental view postulates that Q is the sole fundamental determinant of investment in a causal sense.[18] The strong form shares the same empirical hypothesis of the semi-strong form, but they differ in their interpretations. The strong form of the fundamental view would interpret the non-rejection of the null hypothesis as a causal relation between investment and Q, while the semi-strong form would interpret just as a mere correlation.[19]

A special case of the strong form, originating from Tobin and Brainard (1976) and implicitly assuming no adjustment costs, believes that Q should equal one at equilibrium and that $Q > 1$ should stimulate investment and $Q < 1$ discourages investment.[20] We refer to this view as the "*unity view*".[21] As noted earlier, it is technically implausible to have both CRS for convenient measurement of Q and zero adjustment costs for simplicity. Theoretically, moreover, models without adjustment costs have unrealistic quantitative implications for asset prices.[22] Therefore, we do not discuss further about the unity view, which is inconsistent with the modern formulation of Q theory.

Unfortunately, empirical results are at odds with the fundamental view at all levels. Investment and Q frequently diverge both in the aggregate economy and across many industries.[23] In Figure **??**, investment and Q move in opposite directions with a frequency of 56% in the

[17]See A. B. Abel (1980), Summers et al. (1981) for early contributions and Hassett and Hubbard (1997) and Caballero (1999) for early literature reviews. Whited (1994) argues against relating the OLS coefficient to the inverse of C because the regression equation can be integrated to a whole class of adjustment cost functions including the quadratic one as a special case.

[18]In a survey on the theory of investment, Chirinko (1993), after deriving q theory, states "For a forward-looking firm constrained by adjustment costs, I_t/K_t should be solely determined by contemporaneous q_t." Summers et al. (1981) also puts, "As Tobin has explained, aggregate investment can be expected to depend in a stable way on q...". Wildasin (1984): "In other words, marginal q is the fundamental determinant of investment..." Fazzari, Hubbard, and Petersen (1988): "...models based on q that emphasize market valuations of the firm's assets as the determinant of investment..."

[19]Cummins et al. (1994) use exogenous tax reforms to examine how investment reacts to exogenous tax-related part of variations in Q

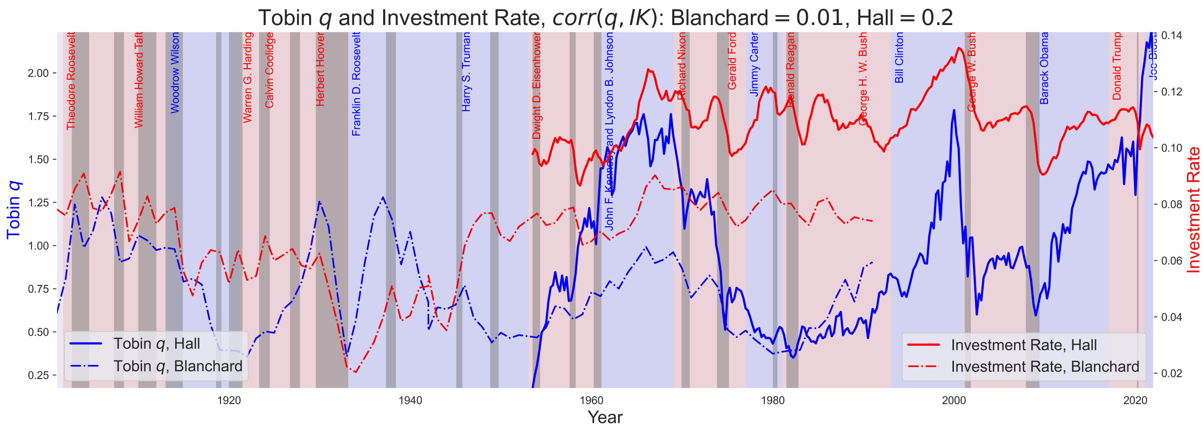
[20]Tobin and Brainard (1976): "Economic logic indicates that a normal equilibrium value for q is 1 for reproducible assets which are in fact being reproduced, and less than 1 for others. Values of q above 1 should stimulate investment, in excess of requirements for replacement and normal growth, and values of q below 1 discourage investment."

[21]For example, Gormsen and Huber (2023) incorporates the corporate discount rate wedge in calculating Tobin's Q so that adjusted Q (Figure 7) closely tracks unity in the time series. In their slides (p.15), they quote Q theory as "Investment should rise until Tobin's $Q = 1$."

[22]As pointed out by Kogan and Papanikolaou (2012), without adjustment costs: "The smooth process for the price of capital is at odds with the data, where the market value of capital is much more volatile than its quantity."

[23]Hassett and Hubbard (1997) documents a low correlation between the level of real investment and the level of Q and a negative correlation between their growth rates. Blanchard, Rhee, and Summers (1993) also reports that the investment-Q relation is not tight at low frequencies.

Figure 2: Aggregate Investment rate and average Q based on Blanchard, Rhee, and Summers (1993) and Hall (2001)



Note: The dash lines represent data scraped from Blanchard, Rhee, and Summers (1993) over the period of 1900-1990 and the solid lines represent data over the period of 1953 to 2021 constructed following the methodology of Hall (2001).

sample of Blanchard, Rhee, and Summers (1993) spanning over 1900-1990 and of 51% in the sample that is constructed following Hall (2001) and covers the whole post-war period until the COVID crisis. In addition, only 2 out of 20 4-digit GICS industries (excluding financial services broadly) exhibits positive correlation between investment and Q. Investment-Q regression has low R^2 with other explanatory variables (e.g., cash flows) being statistically significant, and the slope estimate is unrealistically small, implying implausibly high adjustment costs.[24]

Under this fundamental view, many subsequent studies argue that marginal Q, once properly measured, should still be the sufficient statistic for investment. Empirical studies fail to uncover a satisfactory relation between investment and Q because many auxiliary assumptions of Q-theory model are inconsistent with data. For example, average Q exceeds marginal Q when firms enjoy monopoly rents due to DRS technology or imperfect competition (Lindenberg and Ross, 1981). Average Q fails to capture marginal Q for the specific type of capital when capital goods are heterogeneous and cannot be simply summed up (Hayashi and Inoue, 1992). Average Q overestimates marginal Q for physical capital when firm values are derived from intangible capital (Hall, 2001; Peters and Taylor, 2017). Average Q can also differ from true marginal Q in presence of empirical measurement errors (Erickson and Whited, 2000, 2006, 2011). In other words, Q-theory does not fail, but those overly simplistic auxiliary assumptions fail, rendering Q-theory barely empirically testable.

[24] See Summers et al. (1981) and Fazzari, Hubbard, and Petersen (1988) for early contributions. Philippon (2009): "The investment equation fits poorly, leaves large unexplained residuals correlated with cash flows, and implies implausible parameters for the adjustment cost function."

This paper argues that, in presence of supply shocks to investment, there is no longer a one-to-one relation between investment and Q, even when marginal Q can be precisely measured.

3 Equilibrium View

The fundamental view is mainly based on the equation (**??**), which is derived from the original optimality condition (**??**) of the value-maximizing problem of the firm under assumptions of constant returns-to-scale and quadratic adjustment costs. To interpret Q theory properly, one should take the optimality condition (**??**) verbatim, that is, the firm should invest until the marginal cost equals the marginal benefit of investment. Alternatively, the optimal investment policy equates the marginal benefit and cost of investment. Therefore, investment and Q are simultaneously determined in equilibrium by exogenous forces. The joint equilibrium determination of investment and Q is the starting point and foundation of the *equilibrium view*.

Analogous to quantity and price, investment and Q are jointly determined by investment demand and investment supply. The inverse investment demand function is given by the schedule of marginal benefit of investment, which is primarily driven by the expected profitability and the discount rate.[25] Higher profitability and lower discount increases the demand for investment. The inverse investment supply function is given by the schedule of marginal cost of investment, which is determined by the cost of investment adjustment. Higher costs of investment translate into lower supply of investment. At each point in time, the intersection of the investment demand and supply yields the equilibrium investment and Q.

To convey the intuition of the equilibrium view and the impact of supply shocks, we first illustrate in a transparent two-period model. Second, in the dynamic setting, we derive propositions under general conditions and discuss the complexity of understanding the joint dynamics of investment and Q.

3.1 Two-period Model

The model setup remains identical except that the firm operates only in period t and $t+1$ and exits in period $t+1$. In period $t+1$, the firm distributes all operating profits Π_{t+1} and the remaining capital to equity holders. We further assume a quadratic adjustment cost and a

[25]As will be clear shortly, the investment demand depends only on the expected profitability and the discount rate in the two-period model and additionally on the expected cost of investment adjustment in the dynamic model. The investment supply remains identical in both settings.

constant discount rate.[26] We have MBI and MCI as follows.

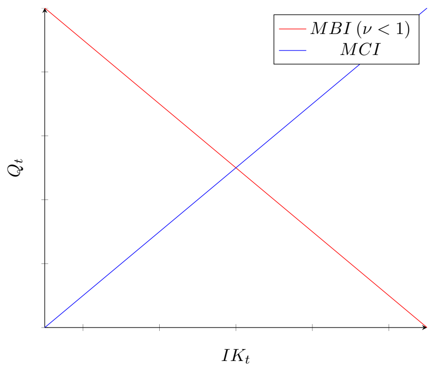
$$MCI_t = 1 + C_t IK_t \tag{16}$$

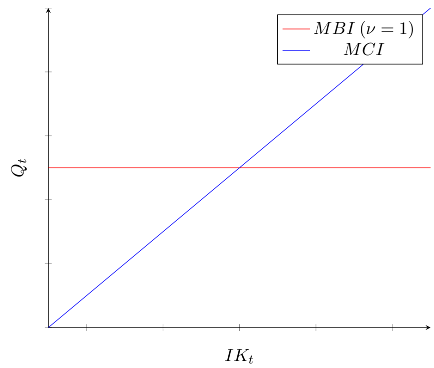
$$Q_t \equiv MBI_t = [\nu \mathbb{E}_t(A_{t+1}^{1-\nu})K_{t+1}^{\nu-1} + (1-\delta)]/R \tag{17}$$

The investment supply, or MCI, is given by the first-order derivative of the adjustment cost function. Here the investment supply is a linear function of investment with the slope C_t. The investment demand, or MBI, is determined by the profit function, the expectation and the discounting. All else equal, the slope of the investment demand depends the degree of returns to scale. The investment demand, $MBI = (\mathbb{E}_t(A) + 1 - \delta)/R$, is invariant to investment under constant returns to scale ($\nu = 1$), whereas the investment demand is downward sloping under decreasing returns to scale ($\nu < 1$).

We now graphically illustrate the investment demand and supply curve. First note that both MBI and MCI are functions of the investment rate, i.e., $Q_t = Q(IK_t) = MBI(IK_t)$ and $MCI_t = MCI(IK_t)$.[27] To clarify, Q is the marginal benefit of investment by definition, whereas Q equals the marginal cost of investment only at optimum. Therefore, we denote IK^* as the optimal level of investment, i.e., $MCI(IK^*) = MBI(IK^*)$, and Q^* as MBI at the optimal level of investment, i.e., $Q^* \equiv Q(IK^*) = MBI(IK^*)$. Distinguishing Q, the function, from Q^*, the optimal value, clears up a lot of confusion around interpretations of Q theory.

Figure 3: Investment Demand and Supply in a Two-period Model





Note: this figure illustrates the marginal cost of investment (i.e., investment supply schedule) and the marginal benefit of investment (i.e, investment demand schedule) in the simple two-period model. The left panel is under the condition of decreasing returns-to-scale operating profits. The right panel is under the condition of constant returns-to-scale operating profits. The marginal cost of investment remains the same in both panels.

[26] By assuming constant discount rate, we are unable to discuss any asset pricing implications of Q theory. The focus here is on the optimal investment and Q in equilibrium. Writing a similar two-period model, Zhang (2017) provides an excellent discussion of asset returns in relation to Q theory.

[27] Since K_t is fixed at time t, it is innocuous to refer to either the level of investment or the investment rate in most context. Also, once investment completed, K_{t+1} is deterministic at date t, and it is again innocuous to refer to either I_t or K_{t+1} as the choice variable.

The intersection of the investment demand and supply curves delivers the optimal investment IK^* and Q^*. Suboptimal investment reduces the firm value. When the firm invests below the optimal level IK^*, $Q(IK)$ is higher than $MCI(IK)$, regardless of the degree of returns to scale (ν). Underinvestment depresses firm value as the firm does not fully capture available profitable investment opportunities. Similarly, when the firm invests above the optimal level IK^*, $Q(IK)$ is lower than $MCI(IK)$. Overinvestment destroys firm value as marginal costs exceeds marginal benefits in those additional investment projects.

Table 1: Investment demand and supply shift, impacting equilibrium investment and Q

	Investment Demand		Investment Supply	
	Upward	Downward	Upward	Downward
Q^*	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
IK^*	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$Cov(Q^*, IK^*)$	$+$	$+$	$-$	$-$

Note: this table illustrates the direction of movement of investment and Q given shifts of the investment demand curve and shifts of the investment supply curve.

For a given set of parameter values, IK^* and Q^* are uniquely determined by the investment demand and supply curves. Changes of parameter values shift the investment demand and supply and result in different values of IK^* and q^*. For example, a higher profitability makes the investment demand steeper when $\nu < 1$ and shifts the investment demand upward when $\nu = 1$. A larger discount rate makes the investment demand flatter when $\nu < 1$ and shifts the investment demand downward when $\nu = 1$. A larger C_t makes the investment supply steeper. When the investment demand shifts upward (downward), both IK^* and Q^* will increase (decrease). When the investment supply shifts upward (downward), IK^* will decrease (increase) while Q^* will increase (decrease). Steepening (flattening) the curve has the same qualitative effects as upward (downward) shifting. In short, IK^* and Q^* move in the same direction when the investment demand shifts, and they diverge when the investment supply shifts. When both curves shift, one of IK^* and Q^* will move unambiguously while the other is determined by net effects of both curves' movement. The effects of curve shifting are summarized in Table **??**.

3.2 Dynamic Model

The equilibrium analysis in this static model is straightforward as shifters of one curve does not shift the other. In the dynamic setting, however, supply shocks not only shift the investment supply curve but also potentially shift the investment demand curve.

Assuming a constant discount rate, we have

$$MCI_t = 1 + C_t IK_t^{\eta} \tag{18}$$

$$MBI_t = \frac{1}{R}\mathbb{E}_t\Big[\Big(\nu A_{t+1}^{1-\nu} K_{t+1}^{\nu-1} + \frac{\eta C_{t+1}}{\eta+1} IK_{t+1}^{\eta+1} + (1-\delta)\big(1 + C_{t+1} IK_{t+1}^{\eta}\big)\Big)\Big] \quad (0 < \nu < 1) \tag{19}$$

$$MBI_t = \frac{1}{R}\mathbb{E}_t\Big[\Big(A_{t+1} + \frac{\eta C_{t+1}}{\eta+1} IK_{t+1}^{\eta+1} + (1-\delta)\big(1 + C_{t+1} IK_{t+1}^{\eta}\big)\Big)\Big] \quad (\nu = 1) \tag{20}$$

Very roughly speaking, holding all else equal, a positive shock to the expected profitability, increases investment as well as Q. In contrast, holding all else equal, a positive shock to investment costs C_t, decreases investment and may increase or decrease Q.

There is a danger of invoking "ceteris paribus" in the dynamic model. Arguments above properly highlight the potential role of supply shocks in driving the joint dynamics of investment and Q. However, they are improper in a dynamic equilibrium model since one cannot really vary a particular variable while fixing all other variables, especially those endogenous ones. In a dynamic model, MBI not only is affected by today's investment but also involves tomorrow's optimal investment decision. Today's investment could potentially affect tomorrow's investment under some conditions, for example, DRS ($0 < \nu < 1$). Thus, it is unclear whether today's supply shocks will change tomorrow's marginal payoff, therefore MBI. The analysis is complicated additionally by the fact that MBI also involves tomorrow's investment costs. Shocks to today's investment costs could potentially affect MBI under some conditions, for example, if C_t are persistent.

Despite the complexity, we can still derive some properties of investment and Q from the general case without explicitly solving the model.

Regardless of CRS or DRS, the following condition always holds true at optimum

$$Q_t^* = 1 + C_t IK_t^{*\eta} \tag{21}$$

where $Q_t^* > 1$ denotes the equilibrium marginal Q, and IK_t^* denotes the equilibrium investment rate. To understand how demand and supply shocks affect the joint dynamics of investment and Q, we take derivative of both sides with respect to demand and supply shocks.

$$\beta_t^Q := \frac{\partial(Q_t^* - 1)/(Q_t^* - 1)}{\partial A_t/A_t} = \eta \frac{\partial IK_t^*/IK_t^*}{\partial A_t/A_t} =: \eta\beta_t \tag{22}$$

$$\phi_t^Q := \frac{\partial(Q_t^* - 1)/(Q_t^* - 1)}{\partial C_t/C_t} = 1 + \eta \frac{\partial IK_t^*/IK_t^*}{\partial C_t/C_t} =: 1 + \eta\phi_t \tag{23}$$

where, for convenience, we denote the elasticity of investment and $Q^* - 1$ to demand shocks as β_t and β_t^Q, respectively, and denote the the elasticity of investment and $Q^* - 1$ to supply shocks as ϕ_t and ϕ_t^Q. Elasticities of $Q^* - 1$ to demand and supply shocks linearly related to elasticities of IK^* to demand and supply shocks. Specifically, we have following propositions.

Proposition 1. *$\beta_t^Q = \eta\beta_t$. β_t^Q is increasing in β_t for $\eta > 0$.*

$$\begin{cases} \beta_t^Q > 0, & if \quad \beta_t > 0 \\ \beta_t^Q = 0, & if \quad \beta_t = 0 \\ \beta_t^Q < 0, & if \quad \beta_t < 0 \end{cases}$$

When A_t changes, Q_t^* and IK_t^* always move in the same direction if $\beta_t \neq 0$. Both Q_t^* and IK_t^* are invariant to C_t if $\beta_t = 0$.

Proposition **??** shows that, all else equal, given only demand shocks A_t, Q_t^* and IK_t^* will always move in the same direction as long as the elasticity of investment to demand shocks is non-zero. When the elasticity of investment to demand shocks is zero, both Q_t^* and IK_t^* are invariant to demand shocks. Intuitively, the elasticity of investment to demand shocks is zero if demand shocks are i.i.d. because realized demand shocks today are uninformative about demand shocks tomorrow. Proposition **??** implies that, in presence of only demand shocks, Q_t^* and IK_t^* are positively correlated (the correlation coefficient of Q_t^* and IK_t^* that are both constant can be interpreted as one). Under what conditions are they perfectly positively correlated?

Corollary 1. *In presence of only demand shocks (A_t is stochastic and C_t is constant), the Spearman's rank correlation coefficient between Q_t^* and IK_t^* is always $+1$ for $\eta > 0$, and the Pearson's correlation coefficient between Q_t^* and IK_t^* is strictly $+1$ if and only if $\eta = 1$.*

Intuitively, with only demand shocks, the investment supply curve remains identical over time. No matter how the investment demand curve shifts over time, the equilibrium Q_t^* and IK_t^* will always show up on the same upward sloping investment supply curve. Thus, their

Spearman's rank correlation coefficient will always equals one regardless of the curvature of the investment supply curve, and their Pearson's correlation coefficient will only equal one when the investment supply curve is linear, i.e., when $\eta = 1$.

We now turn to the case with supply shocks.

Proposition 2. $\phi_t^Q = 1 + \eta\phi_t$. *$\phi_t^Q$ is increasing in ϕ_t if $\eta > 0$.*

$$\begin{cases} \phi_t^Q > 0, & if \quad \phi_t > -\frac{1}{\eta} \\ \phi_t^Q = 0, & if \quad \phi_t = -\frac{1}{\eta} \\ \phi_t^Q < 0, & if \quad \phi_t < -\frac{1}{\eta} \end{cases}$$

When C_t changes, Q_t^* and IK_t^* move in the same direction if $\phi_t \in (-\infty, -\frac{1}{\eta}) \cup (0, +\infty)$, and move in the opposite direction if $\phi_t \in (-\frac{1}{\eta}, 0)$. IK_t^* is invariant to C_t if $\phi_t = 0$, and Q_t^* is invariant to C_t if $\phi_t = -\frac{1}{\eta}$.

Proposition **??** shows that, in response to changes in C_t, Q_t^* and IK_t^* move in the opposite direction if the elasticity of investment to the adjustment cost scalar C_t is negative and sufficiently inelastic. The intuition is straightforward. $Q_t^* - 1$ is a product of C_t and $IK_t^{*\eta}$. When C_t increases, Q_t^* will increase holding IK^* fixed. If IK_t^* increases in C_t, then Q_t^* will certainly increase, regardless of the magnitude of its elasticity. If IK_t^* decreases very elastically in C_t, Q_t^* will also decrease. Only if IK_t^* decreases inelastically in C_t will Q_t^* and IK_t^* move in opposite direction. The threshold for the elasticity is $-\frac{1}{\eta}$. Proposition **??** implies that, in presence of only supply shocks, Q_t^* and IK_t^* could be potentially correlated to various extent, depending on the elasticity of investment to supply shocks, in contrast to the case with only demand shocks in which Q_t^* and IK_t^* could only be positively correlated.

Corollary 2. *In presence of only supply shocks (A_t is constant and C_t is stochastic), the Pearson's correlation coefficient between Q_t^* and IK_t^* is strictly ± 1 if and only if $\phi_t = \frac{-1}{\eta - 1}$ and $\eta \neq 1$. Q_t^* and IK_t^* are uncorrelated if and only if $\phi_t = 0$ or $\phi_t = \frac{-1}{\eta}$.*

Proof. Q_t^* and IK_t^* are perfectly correlated if $Q_t^* - 1$ and IK_t^* are so. $Q_t^* - 1$ and IK_t^* are perfectly correlated if it can be written as $Q_t^* - 1 = a + bIK_t^*$ for some constants a, b. Recall that $Q_t^* - 1 = C_t IK_t^{*\eta}$ has no constant terms, thus $a = 0$. $Q_t^* - 1$ and IK_t^* are perfectly correlated if $bIK_t^* = C_t IK_t^{*\eta}$ holds true for some constant b. The condition requires $IK_t^* = (C_t/b)^{\frac{-1}{\eta-1}}$ and $\eta \neq 1$, which implies the elasticity of investment to C_t is constant and equals $\phi_t = \frac{-1}{\eta-1}$.

It is easy to check that $\phi_t^Q = 1 + \eta\phi_t = \phi_t = \frac{-1}{\eta-1}$. $Q_t^* - 1$ and IK_t^* are perfectly positively (negatively) correlated if $b > 0$ ($b < 0$). In a system with only one source of shocks, Q_t^* and IK_t^* are uncorrelated if and only if $\frac{\partial IK_t^*}{\partial C_t} = 0$ or $\frac{\partial Q_t^*}{\partial C_t} = 0$, equivalent to $\phi_t = 0$ or $\phi_t^Q = 0$, which correspond to $\phi_t = 0$ or $\phi_t = \frac{-1}{\eta}$. □

Corollary **??** establishes the sufficient and necessary condition for the perfect correlation and uncorrelation between Q_t^* and IK_t^* in presence of only supply shocks. However, note that $b < 0$ implies $C_t < 0$ and $Q^* < 1$ in a model with only positive investment (as the aggregate investment). It suggests that a perfect negative correlation requires negative adjustment cost scalar C_t in addition to $\phi_t = \frac{-1}{\eta-1}$ and $\eta \neq 1$

What determines the elasticity of investment to profitability A_t and adjustment cost C_t? To answer this question, we can solve the model with CRS technology using log-linearization. It turns out that the elasticity of investment to demand and supply shocks crucially depends their persistence.

4 The Case of CRS: Analytical Solution

The goal of this section is to analyze elasticities of the investment rate explicitly using a first-order approximation. To this end, we log-linearize the model and solve the optimal investment rate with an approximate explicit solution. The solution method follows Belo and Li (2023).

Assuming CRS ($\nu = 1$) and a constant discount rate R, we have

$$1 + C_t IK_t^\eta = \frac{1}{R}\mathbb{E}_t\left[A_{t+1} + \frac{\eta}{\eta+1}C_{t+1}IK_{t+1}^{\eta+1} + (1-\delta)\left[1 + C_{t+1}IK_{t+1}^\eta\right]\right] \tag{24}$$

Alternatively, the optimality condition can be rewritten as that the firm invests until the expected investment return equals the discount rate.

$$R = \mathbb{E}_t\left[\frac{A_{t+1} + \frac{\eta}{\eta+1}C_{t+1}IK_{t+1}^{\eta+1} + (1-\delta)\left[1 + C_{t+1}IK_{t+1}^\eta\right]}{1 + C_t IK_t^\eta}\right] \equiv \mathbb{E}_t[R_{t+1}^I] \tag{25}$$

We log-linearize the investment return with respect to the investment rate (ik), the profitability (a), and the adjustment cost scalar (c) around their non-stochastic steady-state values. Note that the non-stochastic steady state investment rate is $\overline{IK} = \delta$, which is the fixed point of the law of motion for capital.

$$
\begin{aligned}
r^I_{t+1} &\equiv \log(R^I_{t+1}) = \log(MPI_{t+1}) - \log(MCI_t) \\
&= \log(A_{t+1} + \frac{\eta}{\eta+1} C_{t+1} IK^{\eta+1}_{t+1} + (1-\delta)\big[1 + C_{t+1} IK^{\eta}_{t+1}\big]) - \log(1 + C_t IK^{\eta}_t)
\end{aligned} \tag{26}
$$

For the first term, we have

$$
\begin{aligned}
\log(MPI_{t+1}) \approx& \log(\overline{MPI}) + \frac{e^{\overline{a}}}{e^{\overline{a}} + \frac{\eta}{\eta+1} e^{\overline{c}+(\eta+1)\overline{ik}} + (1-\delta)\big(1 + e^{\overline{c}+\eta\overline{ik}}\big)} (a_{t+1} - \overline{a}) \\
&+ \frac{e^{\overline{c}+\eta\overline{ik}}\big(\frac{\eta}{\eta+1}\delta + 1 - \delta\big)}{e^{\overline{a}} + \frac{\eta}{\eta+1} e^{\overline{c}+(\eta+1)\overline{ik}} + (1-\delta)\big(1 + e^{\overline{c}+\eta\overline{ik}}\big)} (c_{t+1} - \overline{c}) \\
&+ \frac{\eta e^{\overline{c}+\eta\overline{ik}}}{e^{\overline{a}} + \frac{\eta}{\eta+1} e^{\overline{c}+(\eta+1)\overline{ik}} + (1-\delta)\big(1 + e^{\overline{c}+\eta\overline{ik}}\big)} (ik_{t+1} - \overline{ik}) \\
&\equiv \gamma_1 + \lambda_1 \overline{ik}_{t+1} + \theta_1 \overline{c}_{t+1} + \omega_1 \overline{a}_{t+1}
\end{aligned}
$$

where $\log(\overline{MPI}) = \log(e^{\overline{a}} + \frac{\eta}{\eta+1} e^{\overline{c}+(\eta+1)\overline{ik}} + (1-\delta)\big(1 + e^{\overline{c}+\eta\overline{ik}}\big)) \equiv \gamma_1$ is the steady-state value of the marginal payoff of investment, $\overline{ik}_{t+1} \equiv ik_{t+1} - \overline{ik}$ is the deviation of the log investment rate from its steady-state value, and other overlined variables denote their demeaned counterparts.

For the second term, we have

$$
\begin{aligned}
\log(MCI_t) \approx& \log(\overline{MCI}) + \frac{e^{\overline{c}+\eta\overline{ik}}}{1 + e^{\overline{c}+\eta\overline{ik}}} (c_t - \overline{c}) + \frac{\eta e^{\overline{c}+\eta\overline{ik}}}{1 + e^{\overline{c}+\eta\overline{ik}}} (ik_t - \overline{ik}) \\
\equiv& \gamma_2 + \lambda_2 \overline{ik}_t + \theta_2 \overline{c}_t
\end{aligned}
$$

where $\log(\overline{MCI}) = \log(1 + e^{\overline{c}+\eta\overline{ik}}) \equiv \gamma_2$ is the steady-state value of the marginal cost of investment. The steady-state return is given by $\overline{R} = \frac{\overline{MPI}}{\overline{MCI}}$.

Finally, we have the log-linearized investment return as follows.

$$
\begin{aligned}
r^I_{t+1} &\approx (\gamma_1 + \lambda_1 \overline{ik}_{t+1} + \theta_1 \overline{c}_{t+1} + \omega_1 \overline{a}_{t+1}) - (\gamma_2 + \lambda_2 \overline{ik}_t + \theta_2 \overline{c}_t) \\
&= (\gamma_1 - \gamma_2) + (\lambda_1 \overline{ik}_{t+1} - \lambda_2 \overline{ik}_t) + (\theta_1 \overline{c}_{t+1} - \theta_2 \overline{c}_t) + \omega_1 \overline{a}_{t+1}
\end{aligned} \tag{27}
$$

Now we conjecture the optimal investment rate as follows.

$$
\overline{ik}_t = \alpha + \beta \overline{a}_t + \phi \overline{c}_t \tag{28}
$$

where α is a constant, β is the elasticity of the investment rate to the profitability, and ϕ is the elasticity of the investment rate to the adjustment cost scalar.

We now specify dynamics of exogenous state variables.

$$\overline{a}_{t+1} = \rho_a \overline{a}_t + \sigma_a e_{a,t+1} \tag{29}$$

$$\overline{c}_{t+1} = \rho_c \overline{c}_t + \sigma_c e_{c,t+1} \tag{30}$$

where $\bar{a},\ \bar{c}$ are unconditional means, and $e_{a,t+1}, e_{c,t+1} \overset{i.i.d}{\sim} N(0,1)$ are orthogonal to each other.

Substituting in the optimal investment rate, we have the log investment return as follows.

$$\begin{aligned}
r^I_{t+1} =& (\gamma_1 - \gamma_2) + [\lambda_1 \overline{ik}_{t+1} - \lambda_2 \overline{ik}_t] + (\theta_1 \overline{c}_{t+1} - \theta_2 \overline{c}_t) + \omega_1 \overline{a}_{t+1} \\
=& (\gamma_1 - \gamma_2) + \lambda_1(\alpha + \beta \overline{a}_{t+1} + \phi \overline{c}_{t+1}) - \lambda_2(\alpha + \beta \overline{a}_t + \phi \overline{c}_t) + (\theta_1 \overline{c}_{t+1} - \theta_2 \overline{c}_t) + \omega_1 \overline{a}_{t+1} \\
=& [(\gamma_1 - \gamma_2) + (\lambda_1 - \lambda_2)\alpha] + (\lambda_1 \beta + \omega_1)\overline{a}_{t+1} - (\lambda_2 \beta)\overline{a}_t + (\lambda_1 \phi + \theta_1)\overline{c}_{t+1} - (\lambda_2 \phi + \theta_2)\overline{c}_t \\
=& [(\gamma_1 - \gamma_2) + (\lambda_1 - \lambda_2)\alpha] + [(\lambda_1 \beta + \omega_1)\rho_a - \lambda_2 \beta]\overline{a}_t + [(\lambda_1 \phi + \theta_1)\rho_c - (\lambda_2 \phi + \theta_2)]\overline{c}_t \\
& + (\lambda_1 \beta + \omega_1)\sigma_a e_{a,t+1} + (\lambda_1 \phi + \theta_1)\sigma_c e_{c,t+1} \\
\equiv& \mu_r + \rho^r_a \overline{a}_t + \rho^r_c \overline{c}_t + \lambda^r_a \sigma_a e_{a,t+1} + \lambda^r_c \sigma_c e_{c,t+1}
\end{aligned}$$

The optimality condition in logarithm is given by

$$r = \mathbb{E}_t[r^I_{t+1}] + \frac{1}{2}\mathbb{V}_t[r^I_{t+1}] \tag{31}$$

which yields

$$\begin{aligned}
r =& [(\gamma_1 - \gamma_2) + (\lambda_1 - \lambda_2)\alpha] + [(\lambda_1 \beta + \omega_1)\rho_a - \lambda_2 \beta]\overline{a}_t + [(\lambda_1 \phi + \theta_1)\rho_c - (\lambda_2 \phi + \theta_2)]\overline{c}_t \\
& + \frac{1}{2}(\lambda_1 \beta + \omega_1)^2 \sigma_a^2 + \frac{1}{2}(\lambda_1 \phi + \theta_1)^2 \sigma_c^2
\end{aligned}$$

In equilibrium, the optimality condition must hold at each point in time. Therefore, it requires

$$\begin{aligned}
0 &= -r + [(\gamma_1 - \gamma_2) + (\lambda_1 - \lambda_2)\alpha] + \frac{1}{2}(\lambda_1 \beta + \omega_1)^2 \sigma_a^2 + \frac{1}{2}(\lambda_1 \phi + \theta_1)^2 \sigma_c^2 \\
0 &= [(\lambda_1 \beta + \omega_1)\rho_a - \lambda_2 \beta] \\
0 &= [(\lambda_1 \phi + \theta_1)\rho_c - (\lambda_2 \phi + \theta_2)]
\end{aligned}$$

from which we obtain elasticities of the investment rate

$$\alpha = \frac{r - (\gamma_1 - \gamma_2) - \frac{1}{2}(\lambda_1\beta + \omega_1)^2\sigma_a^2 - \frac{1}{2}(\lambda_1\phi + \theta_1)^2\sigma_c^2}{(\lambda_1 - \lambda_2)} \tag{32}$$

$$\beta = \frac{-\omega_1\rho_a}{(\lambda_1\rho_a - \lambda_2)} \tag{33}$$

$$\phi = \frac{-(\theta_1\rho_c - \theta_2)}{(\lambda_1\rho_c - \lambda_2)} \tag{34}$$

The solution shows that elasticities of the investment rate are closely related to the persistence of exogenous state variables. We can further analyze how elasticities vary with the persistence.

Lemma 1. *For $\rho_a \in [0, 1]$, $\beta \in [0, \frac{-\omega_1}{(\lambda_1-\lambda_2)}]$ is non-negative and increasing in ρ_a.*

Proof. First, focus on the function $f(\rho) = \lambda_1\rho - \lambda_2$, which is the denominator of elasticities. Since $\lambda_1 > 0$, $f(\rho)$ is increasing in ρ. For $\rho \in [0, 1]$, $f(\rho) \in [-\lambda_2, \lambda_1 - \lambda_2]$, where $\lambda_1 - \lambda_2 < 0$. To see the sign, note that λ_1 and λ_2 share the same numerator, while the denominator of λ_1, $\overline{MPI}$, is larger than that of λ_2, $\overline{MCI}$, as long as the steady-state return is larger than unity, $\overline{R} \equiv \frac{\overline{MPI}}{\overline{MCI}} > 1$. We have $\lambda_1 - \lambda_2 < 0$. In short, $f(\rho) = \lambda_1\rho - \lambda_2$ is increasing in ρ and negative for $\rho \in [0, 1]$, and $1/f(\rho)$ is negative for $\rho \in [0, 1]$ and decreasing in ρ.

For the elasticity of the investment rate to the profitability β, we note that $-\omega_1\rho_a$ is negative and decreasing in ρ_a as $\omega_1 > 0$. Thus, $\beta = \frac{-\omega_1\rho_a}{f(\rho_a)}$ is positive and increasing in ρ_a. For $\rho_a \in [0, 1]$, $\beta \in [0, \frac{-\omega_1}{(\lambda_1-\lambda_2)}]$. β reaches its maximum $\beta_{max} = \frac{-\omega_1}{(\lambda_1-\lambda_2)}$ at $\rho_a = 1$ and its minimum $\beta_{min} = 0$ at $\rho_a = 0$. □

Lemma 2. *For $\rho_c \in [0, 1]$, $\phi \in [\frac{-(\theta_1-\theta_2)}{(\lambda_1-\lambda_2)}, \frac{-\theta_2}{\lambda_2}]$ is negative and decreasing in ρ_c, where $\frac{-\theta_2}{\lambda_2} = -\frac{1}{\eta}$.*

Proof. We first pin down the sign of ϕ. We note that $-(\theta_1\rho_c - \theta_2) > 0$ as $\theta_1 < \theta_2$, because the denominator of θ_1 is larger than that of θ_2 and the numerator of θ_1 is smaller than that of θ_2. Recall that $f(\rho_c) < 0$, so $\phi = \frac{-(\theta_1\rho_c-\theta_2)}{f(\rho_c)} < 0$.

Next we pin down the monotonicity of ϕ. The derivative of ϕ w.r.t. ρ_c, $\frac{\theta_1\lambda_2-\theta_2\lambda_1}{(\lambda_1\rho_c-\lambda_2)^2}$, is negative. First, $\theta_1\lambda_2$ and $\theta_2\lambda_1$ share the same the denominator. Second, The numerator of $\theta_1\lambda_2$ is smaller than that of $\theta_2\lambda_1$ because λ_1 and λ_2 share the same numerator and the numerator of θ_1 is smaller than that of θ_2.

Therefore, $\phi = \frac{-(\theta_1\rho_c-\theta_2)}{f(\rho_c)}$ is negative and decreasing in ρ_c. For $\rho_c \in [0, 1]$, $\phi \in [\frac{-(\theta_1-\theta_2)}{(\lambda_1-\lambda_2)}, \frac{-\theta_2}{\lambda_2}]$, and ϕ reaches its maximum $\phi_{max} = \frac{-\theta_2}{\lambda_2}$ at $\rho_c = 0$ and its minimum $\psi_{min} = \frac{-(\theta_1-\theta_2)}{(\lambda_1-\lambda_2)}$ at $\rho_c = 1$. In addition, we have $\frac{-\theta_2}{\lambda_2} = -\frac{1}{\eta} < 0$. □

Lemma **??** and **??** show that investment is positively related to profitability and negatively related to adjustment costs and that (the absolute value of) elasticities of investment to both profitability shocks and adjustment cost shocks are increasing in their persistence.

We also know how the logarithm of Q_t, or q_t is related to profitability and adjustment costs.

$$
\begin{aligned}
q_t &= [\gamma_2 + \lambda_2 \overline{ik}_t + \theta_2 \overline{c}_t] \\
&= (\gamma_2 + \lambda_2 \alpha) + \lambda_2 \beta \overline{a}_t + (\lambda_2 \phi + \theta_2) \overline{c}_t
\end{aligned} \tag{35}
$$

where $\lambda_2\beta$ is the elasticity of Q_t to the profitability, and $\lambda_2\phi + \theta_2$ is the elasticity of Q_t to the adjustment cost scalar. As β is non-negative and $\lambda_2 > 0$, $\lambda_2\beta$ is non-negative. We have shown that $\phi \leq \frac{-\theta_2}{\lambda_2}$ and, thus, $\lambda_2\phi + \theta_2 \leq 0$. In summary, Q_t is positively related to demand shocks and negatively related to supply shocks.

Now we proceed to understand the correlation between investment and Q. The covariance between investment and Q is related to the covariance between log investment and q in the following way.

$$
COV(IK_t, Q_t) = \mathbb{E}(IK_t)\mathbb{E}(Q_t)(e^{COV(ik_t, q_t)} - 1) \approx \mathbb{E}(IK_t)\mathbb{E}(Q_t)COV(ik_t, q_t) \tag{36}
$$

We can easily write down the the covariance between log investment and q.

$$
COV(ik_t, q_t) = \lambda_2 \beta^2 \mathbb{V}(\overline{a}_t) + (\lambda_2 \phi + \theta_2)\phi \mathbb{V}(\overline{c}_t) \tag{37}
$$

First, note that the unconditional variance of the profitability process $\overline{a}_t$ is given by $\mathbb{V}(\overline{a}_t) = \frac{\sigma_a^2}{1-\rho_a^2}$, so the variance is increasing in the persistence ρ_a. It is obvious that the covariance is monotonically increasing in β, which is increasing in ρ_a. Therefore, the unconditional covariance is increasing in ρ_a.

The relation between the covariance and ρ_c depends $(\lambda_2\phi+\theta_2)\phi\mathbb{V}(\overline{c}_t)$. Focus on the quadratic function, $f(\phi) = (\lambda_2\phi + \theta_2)\psi$, which has two roots, $\psi = 0$ and $\psi = -\frac{1}{\eta}$. It is negative for $\psi \in (-\frac{1}{\eta}, 0)$ and positive otherwise. It decreases on $\psi \in (-\infty, -\frac{1}{2\eta})$. Recall that $\psi \leq -\frac{1}{\eta}$ for $\rho_c \in [0, 1]$, so $f(\phi)$ is positive and decreasing in ϕ. We also know that ϕ is decreasing in $\rho_c \in [0, 1]$. Thus, $f(\phi)$ is increasing in ρ_c and so is $(\lambda_2\phi + \theta_2)\phi\mathbb{V}(\overline{c}_t)$. Therefore, the unconditional covariance is increasing in ρ_c.

We are more interested in the correlation coefficient between investment and Q as it is independent of volatility of two variables. The correlation between investment and Q is closely related to the correlation between investment and Q. In fact, for small values of variances, they are approximately identical.

$$Corr(IK_t, Q_t) = \frac{COV(IK_t, Q_t)}{\sqrt{\mathbb{V}(IK_t)\mathbb{V}(Q_t)}} = \frac{e^{COV(ik_t, q_t)} - 1}{\sqrt{(e^{\mathbb{V}(ik_t)} - 1)(e^{\mathbb{V}(q_t)} - 1)}} \approx \frac{COV(ik_t, q_t)}{\sqrt{\mathbb{V}(ik_t)\mathbb{V}(q_t)}} = Corr(ik_t, q_t) \tag{38}$$

It is analytically difficult, if not entirely impossible, to know how the correlation coefficient varies with respect to parameters.

Lemma **??** shows that, with CRS technology, the elasticity of investment to supply shocks is always no larger than $-\frac{1}{\eta}$. Based on Proposition **??**, we know that, given a supply shock, investment and Q will move in the same direction. Therefore, the case of CRS is of less interest because we want to analyze cases in which investment and Q diverge as frequently observed in real data. This leads us to evaluate the case of DRS technology in the following section.

5 The Case of DRS: Numerical Solution

5.1 Setup

The firm's problem in the case of DRS technology can be formulated as below. Denote $\mathcal{S}_t = (K_t, A_t, C_t)$ as the vector of state variables and $V(\mathcal{S}_t)$, sometimes also denoted by V_t for convenience, as the value function at time t, the Bellman equation of the firm is given by

$$\begin{aligned}
V(\mathcal{S}_t) &= \max_{\{I_t, K_{t+1}\}} \left\{ D_t + \mathbb{E}_t\big[V(\mathcal{S}_{t+1})\big]/R \right\} \\
s.t. \quad D_t &= \mathit{\Pi}_t - I_t - \mathit{\Phi}_t \\
\mathit{\Pi}_t &= A_t^{1-\nu} K_t^{\nu} \\
\mathit{\Phi}_t &= \frac{C_t}{\eta + 1} \left(\frac{I_t}{K_t} \right)^{\eta+1} K_t \\
K_{t+1} &= (1 - \delta) K_t + I_t
\end{aligned}$$

Following Campbell (1994), we specify the exogenous process of technology A_t as follows.

$$A_t = \exp(gt + a_t) \tag{39}$$

$$a_t = (1-\rho_a)\bar{a} + \rho_a a_{t-1} + \sigma_a e_{a,t} \tag{40}$$

where g is the deterministic growth rate of A_t and captures the log-linear time trend in A_t.

We specify the process of adjustment cost scalar C_t as follows.

$$\log(C_t) = c_t = (1-\rho_c)\bar{c} + \rho_c c_{t-1} + \sigma_c e_{c,t} \tag{41}$$

One can easily verify that there exists a balanced growth path in which A_t, K_t, I_t, Φ_t,Π_t, D_t all grow at the same constant rate g. We normalize the economy by removing the time trend to have a stationary problem. Specifically, denote $\hat{K}_t = \frac{K_t}{e^{gt}}$, $\hat{A}_t = \frac{A_t}{e^{gt}}$, $\hat{I}_t = \frac{I_t}{e^{gt}}$, we have

$$V(K_t, A_t, C_t) = \max_{\{I_t, K_{t+1}\}} \left\{ A_t^{1-\nu} K_t^{\nu} - I_t - \frac{C_t}{\eta+1}\left(\frac{I_t}{K_t}\right)^{\eta+1} K_t + \mathbb{E}_t\big[V(K_{t+1}, A_{t+1}, C_{t+1})\big]/R \right\}$$
$$V(K_t, A_t, C_t) = \max_{\{\hat{I}_t, K_{t+1}\}} \left\{ \left(\frac{\hat{A}_t}{\hat{K}_t}\right)^{1-\nu} \hat{K}_t e^{gt} - \hat{I}_t e^{gt} - \frac{C_t}{\eta+1}\left(\frac{\hat{I}_t}{\hat{K}_t}\right)^{\eta+1} \hat{K}_t e^{gt} + \mathbb{E}_t\big[V(K_{t+1}, A_{t+1}, C_{t+1})\big]/R \right\}$$
$$V(\hat{K}_t, \hat{A}_t, C_t) = \max_{\{\hat{I}_t, \hat{K}_{t+1}\}} \left\{ \left(\frac{\hat{A}_t}{\hat{K}_t}\right)^{1-\nu} \hat{K}_t e^{gt} - \hat{I}_t e^{gt} - \frac{C_t}{\eta+1}\left(\frac{\hat{I}_t}{\hat{K}_t}\right)^{\eta+1} \hat{K}_t e^{gt} + \mathbb{E}_t\big[V(\hat{K}_{t+1}, \hat{A}_{t+1}, C_{t+1})\big]/R \right\}$$

Now denote $\hat{V}(\hat{K}_t, \hat{A}_t, C_t) \equiv V(\hat{K}_t, \hat{A}_t, C_t)/e^{gt}$, we can write

$$\hat{V}(\hat{K}_t, \hat{A}_t, C_t)e^{gt} = \max_{\{I_t, K_{t+1}\}} \left\{ \left(\frac{\hat{A}_t}{\hat{K}_t}\right)^{1-\nu} \hat{K}_t e^{gt} - \hat{I}_t e^{gt} - \frac{C_t}{\eta+1}\left(\frac{\hat{I}_t}{\hat{K}_t}\right)^{\eta+1} \hat{K}_t e^{gt} + \frac{\mathbb{E}_t\big[\hat{V}(\hat{K}_{t+1}, \hat{A}_{t+1}, C_{t+1})\big]}{Re^{-g(t+1)}} \right\}$$
$$\hat{V}(\hat{K}_t, \hat{A}_t, C_t) = \max_{\{I_t, K_{t+1}\}} \left\{ \left(\frac{\hat{A}_t}{\hat{K}_t}\right)^{1-\nu} \hat{K}_t - \hat{I}_t - \frac{C_t}{\eta+1}\left(\frac{\hat{I}_t}{\hat{K}_t}\right)^{\eta+1} \hat{K}_t + \frac{\mathbb{E}_t\big[\hat{V}(\hat{K}_{t+1}, \hat{A}_{t+1}, C_{t+1})\big]}{e^{(r-g)}} \right\}$$

which is now a stationary problem and can be solved numerically. The deterministic growth effectively reduces the discount rate by g. We now work with stationary state variables, $k_t = \log(\hat{K}_t)$, $a_t = \log(\hat{A}_t)$, and $c_t = \log(C_t)$, where k_t and a_t correspond to the percentage deviations from the deterministic growth path.

The law of motion for capital can be normalized as follows.

$$
\begin{aligned}
K_{t+1} &= (1-\delta)K_t + I_t \\
\hat{K}_{t+1}e^{g(t+1)} &= (1-\delta)\hat{K}_t e^{gt} + \hat{I}_t e^{gt} \\
\frac{\hat{K}_{t+1}}{\hat{K}_t}e^{g} &= (1-\delta) + \frac{\hat{I}_t}{\hat{K}_t} \\
\hat{IK} &= IK = g + \delta
\end{aligned}
$$

Thus, the steady-state value of investment rate along is given by the sum of growth rate and the depreciation rate $g+\delta$. The steady-state value of capital is derived from the first-order condition of the stationary problem and is given as follows.

$$
\begin{aligned}
R &= \mathbb{E}_t\left[\frac{\nu \hat{AK}_{t+1}^{1-\nu} + \frac{\eta}{\eta+1}C_{t+1}\hat{IK}_t^{\eta+1} + (1-\delta)(1+C_{t+1}\hat{IK}_{t+1}^{\eta})}{1+C_t\hat{IK}_t^{\eta}}\right] \\
\nu \hat{AK}^{1-\nu} &\approx (r+\delta)[1+C(g+\delta)^{\eta}] - \frac{\eta}{\eta+1}C(g+\delta)^{\eta+1}
\end{aligned}
$$

where $R \approx 1+r$, and $\hat{AK}$ denotes the steady-state value of $\hat{A}_t/\hat{K}_t$. Given a value of $\hat{A}$ and C, we can obtain the steady-state value of $\hat{K}$. We choose the unconditional mean of $\hat{A}_t$ and C_t to calculate $\hat{K}$. Note that in the first-order condition of the stationary problem, the LHS should remain as R instead of e^{r-g}.

5.2 Solution Method

The model is solved at the quarterly frequency as most macro models, and model-implied moments are annualized in order to compare with empirical moments. We solve the problem numerically by the method of value function iteration, which is robust and transparent. We discretize k_t, a_t, c_t by the method of Rouwenhorst (1995). Kopecky and Suen (2010) show that the Rouwenhorst method is more robust than the Tauchen (1986) method in approximating highly persistent processes[28]. Specifically, we use 17 grids for a_t, c_t and 301 linear grids for k_t. Given the discrete state space, the conditional expectation is computed as matrix multiplications. Finally, we use a simple global search routine to find the optimal policy in each iteration of value function.

[28] In Tauchen (1986), the author notes that "Experimentation showed that the quality of the approximation remains good except when λ [the serial correlation] is very close to unity." In Tauchen and Hussey (1991), the authors note that for processes with high persistence, "adequate approximation requires successively finer state spaces."

5.3 Calibration

The model is calibrated at annual frequency. The curvature of adjustment costs is set to be $\eta = 1$ to align with the widespread use of quadratic adjustment costs. The annual depreciation rate is set to be $\delta = 0.1$, consistent with the literature. The discount rate is mapped as the real expected returns on assets, which is the weighted average cost of capital. It is set to be $r = 5\%$. The deterministic growth trend is calibrated as $g = 4.35\%$ to match the log-linear trend in the real capital stock of nonfinancial corporate business. The steady-state value of $\hat{K}$ is set to be 1. The operating profit Π_t is mapped to the gross operating surplus. The steady-state value of $\hat{\Pi}/\hat{K}$ is set to be 20% to match the average value of the ratio of gross operating surplus to capital stock. The elasticity of operating profit to capital is set to be $\nu = 0.8$.

5.4 Simulation

The goal is to understand how the correlation between marginal/average Q and investment rate IK varies with the persistence of and the conditional volatility of demand and supply shocks. We simulate exogenous processes of profitability a_t and adjustment cost scalar c_t using continuous multivariate standard normal shocks $e_{a,t+1}$ and $e_{c,t+1}$. Initial values of a_t and c_t are set to be their unconditional means. The initial value of K_t is set to be its steady-state value $\hat{K}$. Given a_t and c_t, we find the optimal investment and the market value using the numerical policy and value function obtained in the solution stage. We interpolate the value function and policy function using 3D linear interpolation during the simulation whenever state variables are off the discrete grids specified in the solution stage. The time series is simulated for 10200 years and the first 200 years are discarded to avoid impacts of initial values.

5.5 Results

Let's first analyze the correlation between marginal Q and investment rates $Corr(MQ, IK)$. Figure **??** shows how $Corr(MQ, IK)$ varies with the level of conditional volatility of adjustment cost shocks σ_c for $\rho_a = 0.95$ and for each level of the conditional volatility of profitability shocks σ_a. First, it is apparent that $Corr(MQ, IK)$ decreases with σ_c. Second, the level of $Corr(MQ, IK)$ increases σ_a is lower. Finally, the speed of reduction in $Corr(MQ, IK)$ w.r.t. σ_c is higher when ρ_c is higher. In summary, the relative volatility of and the persistence of adjustment costs shocks play a crucial role in determining $Corr(MQ, IK)$.

Figure **??** shows how $Corr(MQ, IK)$ varies with the level of conditional volatility of profitability shocks σ_a for $\rho_a = 0.95$ and for each level of the conditional volatility of adjustment costs shocks σ_c. Similar to Figure **??**, we find that $Corr(MQ, IK)$ increases in the volatility of profitability shocks.

Now we analyze the correlation between average Q and investment rates $Corr(MQ, IK)$. Figure **??** shows how $Corr(AQ, IK)$ varies with the level of conditional volatility of adjustment cost shocks σ_c for $\rho_a = 0.95$ and for each level of the conditional volatility of profitability shocks σ_a. It turns out that $Corr(AQ, IK)$ quickly drops below zero as σ_c increases. The pattern is almost homogenous across different levels of ρ_c and across different levels of σ_a, except at small values of σ_c. Figure **??** shows how $Corr(AQ, IK)$ varies with the level of conditional volatility of profitability shocks σ_a for $\rho_a = 0.95$ and for each level of the conditional volatility of adjustment costs shocks σ_c. For smaller values of σ_c, $Corr(AQ, IK)$ also quickly drops below zero as σ_a increases. For higher values of σ_c, however, $Corr(AQ, IK)$ is always negative and increases as σ_a increases. The pattern is also homogenous across different levels of ρ_c. In summary, $Corr(AQ, IK)$ is dominantly determined by the relative volatility of adjustment costs shocks but not by the persistence of adjustment costs shocks.

6 Conclusion

This paper argues for an equilibrium view of Q-theory and carefully analyzes the impact of having investment supply shocks on the relation between investment and Q. The equilibrium view is best summarized as that Q is to investment as price is to quantity in any demand-supply system. Supply shocks can make investment and Q move in opposite directions under some conditions. Specifically, with the DRS technology, the correlation between investment and marginal/average Q critically depends on the relative volatility of and the persistence of supply shocks. While this paper does not provide an empirical approach to identify investment supply shocks, this simple and intuitive theory exercise does provide a new perspective of thinking investment and Q and highlight potential pitfalls underlying the classic Q-theory.

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Figure 4: The correlation coefficient between marginal Q and investment rate with respect to the conditional volatility of adjustment cost shocks σ_c when the persistence of profitability shock $\rho_a = 0.95$, the persistence of adjustment cost shocks $\rho_c = (0.0, 0.2, 0.4, 0.6, 0.8, 0.95)$, the conditional volatility of profitability shocks $\sigma_a = (0.001, 0.01, 0.04, 0.07, 0.1)$

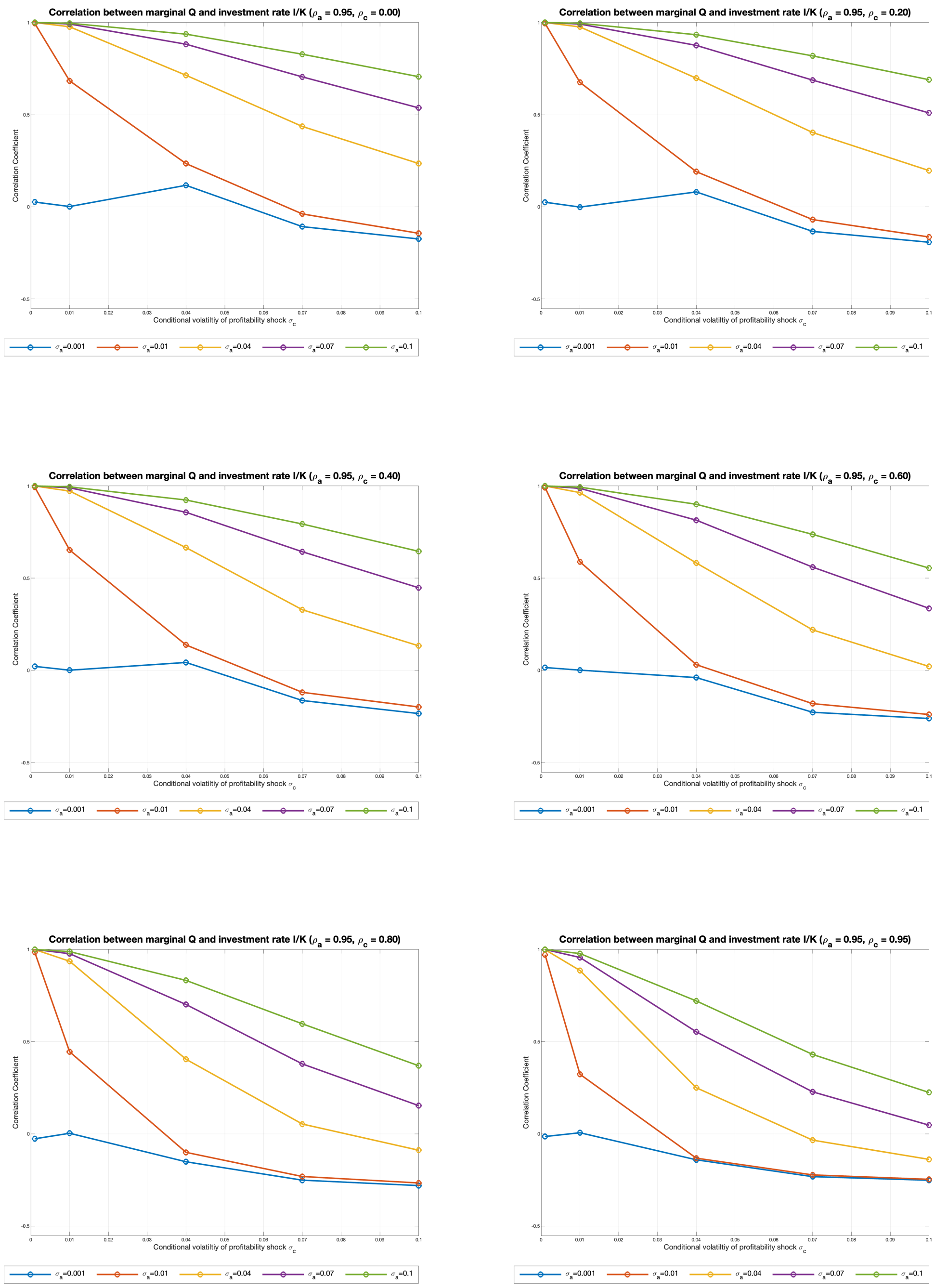

Figure 5: The correlation coefficient between marginal Q and investment rate with respect to the conditional volatility of profitability shocks σ_a when the persistence of profitability shock $\rho_a = 0.95$, the persistence of adjustment cost shocks $\rho_c = (0.0, 0.2, 0.4, 0.6, 0.8, 0.95)$, the conditional volatility of adjustment cost shocks $\sigma_c = (0.001, 0.01, 0.04, 0.07, 0.1)$

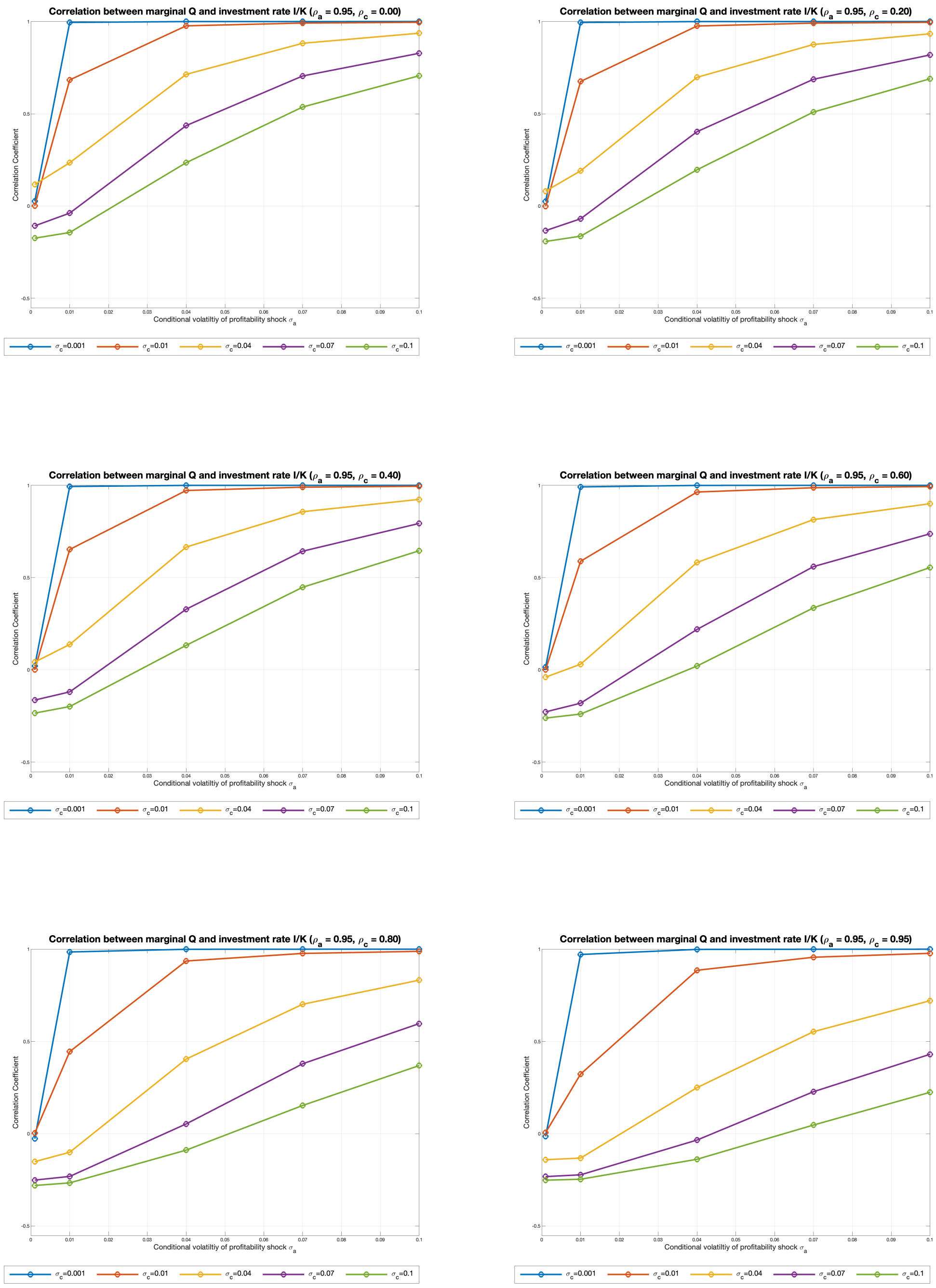

Figure 6: The correlation coefficient between average Q and investment rate with respect to the conditional volatility of adjustment cost shocks σ_c when the persistence of profitability shock $\rho_a = 0.95$, the persistence of adjustment cost shocks $\rho_c = (0.0, 0.2, 0.4, 0.6, 0.8, 0.95)$, the conditional volatility of profitability shocks $\sigma_a = (0.001, 0.01, 0.04, 0.07, 0.1)$

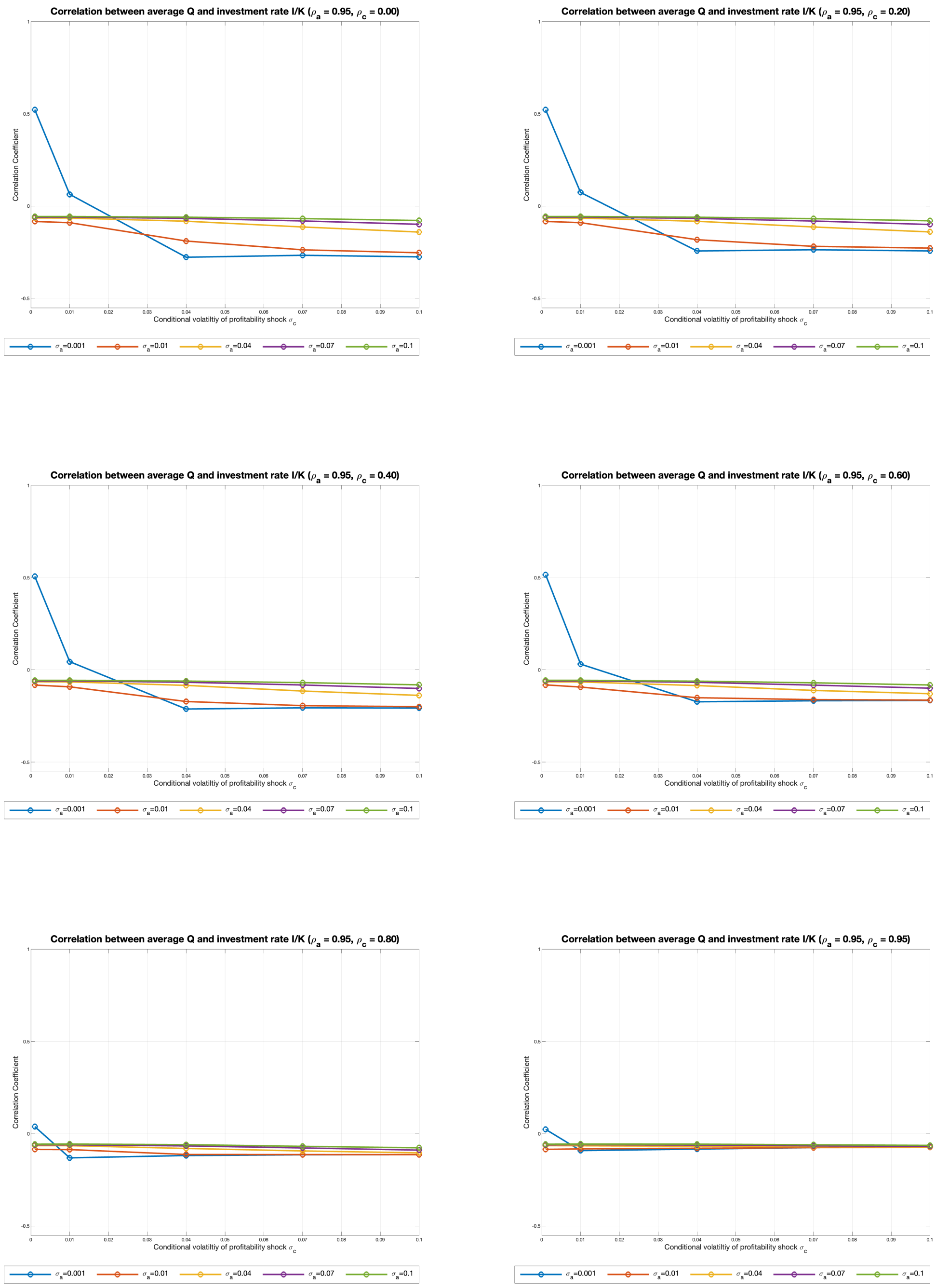

Figure 7: The correlation coefficient between average Q and investment rate with respect to the conditional volatility of profitability shocks σ_a when the persistence of profitability shock $\rho_a = 0.95$, the persistence of adjustment cost shocks $\rho_c = (0.0, 0.2, 0.4, 0.6, 0.8, 0.95)$, the conditional volatility of adjustment cost shocks $\sigma_c = (0.001, 0.01, 0.04, 0.07, 0.1)$